Accounting Conservatism and the Efficient Provision of Capital to Privately Informed Firms

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Abstract

This study addresses conservatism in financial reporting. Firms seek financing for an investment. Investors have access to an imperfect accounting signal that may also have a conservative tendency. If the firms have no private information, conservatism is desirable only for negative \textit{ex ante} expected value investments, consistent with the spirit of the results of Gigler et al (2009) and other studies. Conservatism is not optimal for positive expected value investments because it denies financing to too many of them. If firms have private information, however, conservatism may be desirable even for positive expected value investments. Conservatism is useful in this setting because it increases the probability that privately informed good firms will be able to distinguish themselves from bad firms. Otherwise, good firms prefer not to invest and the financing market fails.
1 Introduction

This study addresses conservatism in financial reporting. Conservatism is usually defined in terms of accounting regulations requiring higher verification standards for profits than for losses. Empirical researchers have documented conservatism in returns-based approaches (see Basu 1997), in which conservatism is measured as the asymmetric timeliness of earnings to bad and good news, and in balance sheet approaches (see Penman and Zhang 2003), in which conservatism is measured as the degree of understatement of assets and income. Researchers have also provided evidence that conservatism is a property of accounting systems globally (Bushman and Piotroski 2006). Watts (2003) surveys common explanations for the prevalence of accounting conservatism: contracting (debt contracting and managerial control), shareholder litigation, political costs, and taxation. This study contributes to the theoretical literature by modeling the role of accounting conservatism in a setting in which firms seeking financing have private information about a project’s expected return. Conservatism is particularly valuable in this setting because it increases the ability of good firms to distinguish themselves from bad firms, thereby preventing failure of the financing market.

In the model, a firm of unknown (good or bad) type seeks financing for a new project. Whether or not the firm knows its type and whether or not the proportion of good firms is high enough that financing transactions can occur without further information, a binary (high or low) accounting signal improves efficiency. One can interpret the signal, which arrives prior to financing and investment, as an earnings report that yields insight into the firm’s ability to generate positive expected value projects. The accounting signal has two properties meant to reflect those of empirically observed accounting systems. First, there is a baseline level of classification error. The baseline error arises because accounting classifications are the product of complex processes of judgment, analysis, and aggregation of information. I assume that that the baseline error is an inherent property of the financial reporting system and not a choice variable. Second, the accounting signal has a tendency, all other things equal, to classify firms as bad, which I label conservatism. Conservatism arises through the agency of an unmodeled regulatory body, and is implemented through institutional means.
such as rules and auditing practices. The most natural interpretation of conservatism in the model is as the inverse of the verification standard for bad news events. That is, as the verification standard becomes lower (easier to satisfy), the overall conservatism of the signal increases. There are more frequent low signal (bad news) realizations, but the conditional probability that the high signal (good news) is accurate increases.

The signal potentially supports a financing equilibrium even when the proportion of good firms is too low to sustain a non-signal equilibrium. The existence of the equilibrium depends on lowering the implicit subsidy on good firms by removing bad firms from the high-signal pool. Lowering the baseline level of error and increasing conservatism both accomplish this goal. If the baseline error rate is low enough, conservatism is not necessary to sustain the equilibrium. Conservatism can substitute for a high baseline error, otherwise. It is not a perfect substitute, as increases in conservatism reduce the frequency of high signals, and therefore the frequency with which good projects receive financing. Under some conditions financing occurs only for high signals. As a result, the inferential properties of the high signal are particularly important. Thus, conservatism may be valuable even if it reduces the overall informativeness of the reporting system.

By influencing the existence and nature of the financing equilibrium, the accounting signal determines the overall expected surplus from investment. In the first-best setting of observable firm type, only good firms receive financing. There are two sources of surplus loss in the second-best pooling equilibrium: good firms that do not receive financing, and bad firms that do.

I consider first the setting in which neither the firm nor the investors know the type. When most firms are good, a conservative accounting signal denies financing to too many good firms and is surplus-destroying. When most firms are bad, however, a conservative accounting signal screens them out and is therefore desirable. Thus, conservatism is surplus-maximizing only when the average project has a negative *ex ante* expected value.

This result is similar to several recent studies focusing on the role of conservatism in mediating financing relationships. Gigler, Kanodia, Sapra, and Venugopalan (2009) model
a setting in which the realization of a public accounting signal potentially triggers a debt covenant-related liquidation of an investment. The optimal debt covenant varies in equilibrium with the properties of the accounting system. The authors find that conservatism is efficient only if the \textit{ex ante} value of the investment is less than its liquidation value. Smith (2007) examines the properties of accounting systems in the context of an investment with real options (staged investment and abandonment). Consistent with Gigler et al (2009), he finds that conservatism is optimal if the \textit{ex ante} investment value is lower than the second stage investment. In Gox and Wagenhofer (2009), the debt contract must motivate the owner/manager of the entity seeking financing to work. Satisfying incentive compatibility may limit the expected cash available for the debt service payment below the level necessary to provide the creditor its required rate of return. In this case, the firm must pledge additional assets. If the expected value of the pledged assets is below that necessary to provide the creditor its return, the firm optimally implements a conservative accounting system that reports (impairs) low-value asset realizations but does not report high-value realizations. The coarseness of the reporting of favorable outcomes maximizes the probability that the creditor provides financing. In all these studies, conservatism in the accounting system is necessary to make a relatively weak \textit{ex ante} investment opportunity viable rather than to enhance the value of a strong one.

Gigler et al (2009) argue that a project’s liquidation value exceeding its \textit{ex ante} expected cash flows, the condition necessary to justify conservatism in their model, is empirically implausible. My study extends the literature by delineating a setting in which conservatism may also be a desirable property for high \textit{ex ante} expected value projects, namely, if the firm has private information about its type. Without private information, high-signal pools of high \textit{ex ante} value projects always receive financing, and it is only the financing of low \textit{ex ante} expected value projects that depends on conservatism. With private information, however, high-signal pools of high-signal projects may not receive financing unless there is a minimum level of conservatism.\footnote{When good-project returns are much higher than the bad-project returns, debt or equity investors can attain first-best by offering financing terms that are acceptable only to good firms. Otherwise, this screening} This result holds because the financing terms must
be incentive compatible to a good firm rather than to an average firm. In the no private information setting, a good firm may realize a negative return if equity investors require a significant dilution of ownership in order to provide financing. The good firm still accepts the financing terms because it does not know its type. The privately informed good firm, however, will not accept these terms, imposing a more stringent informativeness requirement on the high signal. In particular, it may be necessary to winnow out bad firms from the high-signal pool via conservatism in order to reduce the implicit subsidy on the remaining good firms and secure their participation. Thus, my study introduces a plausible tension that allows conservatism to play a larger role in the economy. In particular, conservatism can increase the expected surplus for both high and low ex ante expected value projects.

Researchers have also addressed conservatism in managerial control problems. In some of these studies, conservatism is useful if the ex ante probability of success is relatively low, consistent with the results in Gigler et al (2009), Smith (2007), and the no private information version of my model. For example, Kwon, Newman, and Suh (2001) address the effect of varying conservatism in a principal-agent setting. While their main results pertain to a limited liability setting in which conservatism is optimal, they also show that conservatism is optimal in an unlimited liability setting only if the probability that the desired action generates the high outcome is below $\frac{1}{2}$. In addition, Venugopalan (2001) examines the role of conservatism in an adverse selection model. The manager has private information about the prior probability of project success and chooses an investment level before market exchange occurs. In the fully revealing signaling equilibrium, the manager distorts investment levels to induce a higher signal. If the prior probability is below $\frac{1}{2}$, a maximally conservative signal induces lower investment distortions than a maximally liberal one.\footnote{Because the equilibrium in Venugopalan (2001) is fully revealing, the rational expectations price incorporates all of the manager's information. As a result, the manager judges the market price to be fair, and trade always occurs. In contrast, only the firm in my model knows its type when prices are formed, and trade occurs only if the market price produces a payoff for the good firm at least as high as the payoff from standing} This result and the aforementioned result in Kwon et al (2001) hinge on conservatism

\footnote{2} is not possible. By selling a stake of the firm to investors, for example, the original shareholders forfeit a percentage both of existing assets and future returns. If the future returns are not enough to compensate the original shareholders for their surrendered stake in existing assets, as might be the case if the difference between good and bad returns is not large, the good firm prefers not to seek financing.\footnote{2}
being optimal because it generates a more informative signal if the prior probability of success is low. Conservatism can be optimal in my model even if it undermines the overall informativeness of the reporting system. In another managerial control study, Lin (2006) models a two-period setting in which the principal must induce efficient project selection and appropriate agent effort. A conservative depreciation schedule, replicating the direct communication outcomes, is valuable only if the output from the more profitable project is less informative about effort than the output from the less profitable project. This is loosely analogous to my result that conservatism is useful only if a good project is not too much better than a bad project. Relative to these studies, this study expands the scope of conservatism.

2 The model: equity financing

There are two types of firms, good and bad. The probability that a firm is good type is \( g \), which is common knowledge. Good firms have current value \( V_g \) and bad firms have value \( V_b \), with \( V_g > V_b \). The value is in the form of illiquid assets that will produce a terminal cash flow of \( V_g \) and \( V_b \), respectively, in the next period. Both types of firm have an investment opportunity that they cannot finance internally. For an investment of \( k > 0 \), good firms will realize cash flows of \( k(1 + m_g) \) and bad firms will realize cash flows of \( k(1 + m_b) \), with \( m_g > m_b \) and \( m_g > 0 \). If the firms cannot obtain financing at terms that allow them to break even in expectation given their information, they do not invest and terminal values are \( V_g \) and \( V_b \), respectively. If they can finance the project, the terminal values are \( V_g + k(1 + m_g) \) and \( V_b + k(1 + m_b) \), respectively. The firm seeks financing from risk-neutral equity investors, who require a return of \( r_e \), with \( m_g > r_e > m_b \).

The investors rely on an accounting signal to provide information about the firm’s prospects. One interpretation of the signal is that it is a report on a small existing investment.\(^3\) For example, the investment could be a product trial in a small geographical

\(^3\)By small, I mean sufficiently smaller than the main investment for which the firm seeks funds that it is irrelevant to the subsequent analysis.
area, or preliminary research and development, and the subsequent investment could entail expansion of the project. A more general interpretation is that it is a report of the earnings generated by existing assets, and therefore provides information about the firm’s underlying ability to generate positive expected value projects.

A good firm generates a high signal realization with probability $1 - c + q$, and a low signal otherwise. A bad firm generates a high signal realization with probability $1 - c$, a low signal otherwise. The parameter restriction $0 \leq q \leq c \leq 1$ assures that these are valid probabilities. See Figure 1, Panel A for an illustration of the information structure in the model. I assume that there is no scope for manipulation of the underlying signal by management. Both upward and downward misclassifications can occur. I interpret $q$ as the baseline level of classification accuracy (inverse of classification error) in the accounting system. An increase in $q$ lowers the probability that a good project is misclassified. If $c$ is at its lower bound, an increase in $q$ also leads to an increase in $c$. Thus, $q$ is indirectly related to lowering the probability of misclassifying bad projects. The parameter $c$ is a measure of the conservatism of the accounting system. At the maximum level of conservatism ($c = 1$, Figure 1 Panel B), only a good firm can generate a high signal. At the minimum level of conservatism ($c = q$, Figure 1, Panel C), only a bad project can generate a low signal. Increases in conservatism increase the informativeness of the high signal, but reduce the informativeness of the low signal. The overall effect on the informativeness of the reporting system is ambiguous.\footnote{If $c > \frac{1}{2} + gq$, then the precision of the posterior estimate of type is increasing in conservatism.}

The baseline accuracy, $q$, is an inherent feature of the accounting system, not a choice variable of the regulator who determines accounting rules. Accounting classifications are the product of a complex process involving judgment, estimates, assumptions, and unintentional biases, and are therefore likely subject to ineradicable error. In contrast, the regulator can integrate a conservative bias into the accounting system by, for example, varying the verifiability standards for gains and losses. The natural interpretation of $c$ is as the inverse of the verifiability standard for losses. A lower verifiability standard implies that more projects receive the bad news designation implicit in the low signal. I treat $c$ as a choice variable in
the model in discussions of total surplus from investment. If there were no error in baseline level of error in the accounting system \((q = 1)\), it would constitute a perfect classification system (because \(c\) must also be 1). Given \(q < 1\), the tradeoff between upward and downward classification errors becomes meaningful.

### 2.1 First-best investment

As a preliminary to the main analysis, I characterize the first-best outcome, which occurs if the firm type is observable to investors. Because type is observable, investors anticipate that the terminal value of the good firm, if it seeks financing and invests, is \(V_g + k(1 + m_g)\). If equity investors purchase an ownership stake of \(\alpha\), their payoff is \(\alpha[V_g + k(1+m_g)]\). Since their stake must have a terminal value of \(k(1+r_e)\), the required ownership share is \(\alpha = \frac{k(1+r_e)}{V_g+k(1+m_g)}\).

The liquidating value of the original shareholders’ shares, then, is

\[
V_g + (1+m_g)k = V_g + (m_g - r_e)k.
\]

(1)

The investment increases the value of the original shareholders’ holdings of the good firm by \((m_g - r_e)k\), the profit on the investment net of the cost of equity financing. If the original shareholders do not solicit investment, the terminal value of their stake is \(V_g\). Thus, good firms optimally seek financing in the first-best scenario. Similarly, one can show that investment reduces the value of the original shareholders’ stake of a bad firm to \(V_b - k(r_e - m_b)\) because the cost of capital exceeds the return on the investment. Thus, the bad firm does not invest in the first-best case.

Total expected surplus from investment is:

\[
S = g\pi_g k(m_g - r_e) - (1-g)\pi_b k(r_e - m_b),
\]

(2)

where \(\pi_g\) (\(\pi_b\)) is the probability that a good (bad) firm receives financing. The first-term is the surplus gain from good firms investing. The second term is the surplus loss from bad firms investing. In the first-best setting, total surplus is \(gk(m_g - r_e)\). Implicitly, regulators set the properties of the accounting system to maximize the quantity in equation 2. It will also be useful to define \(g^+ = \frac{r_e - m_b}{m_g - m_b}\) as the proportion of good firms such that the average project (i.e., equation 2 with \(\pi_g = \pi_b = 1\)) has a positive net present value.
In the second-best setting, in which the firm type is unobservable, investors rely on the imperfect accounting signal. Surplus can decline for two reasons. First, pooling may result in financing costs so high that some or all good firms do not seek financing. Second, pooling may result in financing costs low enough that some or all bad firms receive financing.

2.2 Benchmark: firm does not know its type (no information)

As a benchmark, I analyze the case in which the firm does not know its type.

**Proposition 1** If the firm does not know its type, then all firms receive financing if $g > g^+$.

Proof: In the absence of additional information, the expected terminal value is

$$T^\emptyset = g[V_g + k(1 + m_g)] + (1 - g)[V_b + k(1 + m_b)],$$

(3)

implying that investors require a share equal to $\alpha^\emptyset = \frac{k(1 + r_e)}{T^\emptyset}$. If the firm does not accept financing, the terminal value is $gV_g + (1 - g)V_b$. The firm, therefore, accepts financing as long as $(1 - \alpha^I)T^\emptyset > gV_g + (1 - g)V_b$, implying a maximum share sale of $\alpha^I = 1 - \frac{gV_g + (1 - g)V_b}{T^\emptyset}$. Equating $\alpha^\emptyset$ with $\alpha^I$ and solving in terms of $g$ yields $g^+$. QED.

In this setting, the interests of the firm and the investors are aligned. That is, both benefit from a financing transaction if and only if the expected return from the investment exceeds $r_e$, which occurs if the proportion of good firms is high enough. If $g < g^+$, however, financing will not occur without additional information in the form of the accounting signal.

The expected return from a project receiving a high signal is $p_{gh}m_g + (1 - p_{gh})m_b - r_e$, where $p_{gh} = \frac{g(1 - c + q)}{1 - c + gq}$ is the probability the firm is good given a high signal. The expected return for a low signal is $p_{gl}m_g + (1 - p_{gl})m_b - r_e$, where $p_{gl} = \frac{g(c - q)}{c - gq}$ is the probability that the firm is good given a low signal. Let $g_e^{np} = \frac{r_e - m_b}{m_g - m_b - q(m_g - r_e)}$. The following proposition summarizes the financing equilibrium when the firm does not have private information.

**Proposition 2** Assume there is no private information. Then:
i. The first-best scenario in which all and only good firms receive financing cannot be achieved.

ii. If \( g < g^+ \), all high-signal firms receive financing if the baseline level of classification accuracy is high enough (\( q \geq q^{np}_e = \frac{(r_e - m_b) - g(m_g - m_b)}{(1-g)(r_e - m_b)} \)) or the accounting is conservative enough (\( c \geq c^{np}_e = \frac{r_e(1+gq)-(1-g)m_b - g(l+q)m_g}{r_e-(1-g)m_b - gm_g} \)), and the expected surplus is weakly increasing in conservatism. Low-signal firms do not receive financing.

iii. If \( g \geq g^+ \), all high-signal firms receive financing and the expected surplus is maximized by the least conservative accounting \( c = q \).

iv. If \( g \geq g^{np}_e > g^+ \) and the accounting is conservative enough (\( c \geq c^{np}_e = \frac{gq(m_g - r_e)}{g(m_g - m_b) + m_b - r_e} \)), then low-signal firms also receive financing.

The proof is in the Appendix.

Since good and bad firms all have the same beliefs about their type, any investment offer attractive to good firms is also attractive to bad firms. Thus, there is no way to screen out bad firms. The first-best outcome cannot be replicated. The ex ante expected surplus is negative if \( g < g^+ \). Without further information, then, investors are not willing to provide financing to an average firm. The high accounting signal identifies a pool of firms with a higher than ex ante probability of being good. If the classification accuracy, \( q \), is high enough, investors finance firms with high signals regardless of the level of conservatism. Conservatism acts as a substitute for classification accuracy if the accuracy is not high enough (\( q < q^{np}_e \)), removing relatively more bad firms from the high signal pool than good firms. Even if conservatism is not necessary to induce a financing transaction, the expected surplus is increasing in conservatism. The savings from rejecting bad projects exceeds the opportunity cost of rejecting good ones.

The signal is still valuable if the ex ante expected surplus is positive (\( g \geq g^+ \)), though financing would occur without it. Investors finance high-signal firms for all values of classification accuracy and conservatism. If \( g \geq g^+ \), then increasing conservatism removes relatively
more good firms from the high-signal pool than bad ones. The opportunity cost of rejecting good projects exceeds the savings from rejecting bad ones. The expected surplus from high-signal firms, therefore, is decreasing in conservatism.

If \( g > g^{np} \) and the accounting is sufficiently conservative, then there are enough good firms in the low-signal pool that low-signal firms also receive financing. Some degree of conservatism is necessary for this result: if \( c = q \), then all good firms generate high signals, implying that investors will not finance the low-signal pool, which is comprised entirely of bad firms. For high proportions of good firms, conservatism destroys surplus. Setting \( c = q \), guaranteeing that all good firms receive financing, is surplus-maximizing.

Figure 2 Panel A illustrates the results of Proposition 2. For the lowest proportion of good firms \( (g < g^+) \), no financing occurs, implying that surplus is 0, unless conservatism is sufficiently high \( (c > c^{np}_e) \). Beyond that point, surplus is increasing in conservatism. For the middle tranche \( (g^+ \leq g \leq g^{np}_e) \), financing occurs for all and only high-signal firms, and surplus is decreasing in conservatism. For high proportions of good firms \( (g > g^{np}_e) \), surplus is decreasing in conservatism until it is sufficiently high \( (c > c^{np}_e) \) that low-signal firms are also viable candidates for financing. Because all firms are financed beyond that level of conservatism, surplus is fixed, but at a level lower than the surplus induced by the minimum level of conservatism \( (c = q) \).

The results are consistent with Gigler et al (2009), Smith (2007), and Gox and Wagenhofer (2009). Though the settings are different, conservatism is useful in each of these studies only if the investment is \textit{ex ante} not viable. In Gigler et al, conservatism is optimal only if the abandonment value exceeds the \textit{ex ante} expected profit. In Smith (2007), it is optimal only if the second stage investment is higher than the \textit{ex ante} expected terminal cash flow. In Gox and Wagenhofer (2009), conservatism is optimal only if the \textit{ex ante} expected value of the pledged assets is insufficient to satisfy creditor demands. In all cases, conservatism is desirable to improve the viability of a marginal investment opportunity, potentially limiting the role of conservatism in the real economy.
2.3 The firm knows its type

I now assume that the firm knows its type. I first develop the incentive compatibility con-
straints for good and bad firms, which, unlike in the previous section, are different. As
before, the expected terminal price is an average of good and bad firm valuations, consistent
with Bayes’ Rule. For a general expected terminal price $\tilde{P}$ and ownership stake $\alpha$, the equity
investor’s payoff is $\alpha \tilde{P}$. Thus the owners of a good firm must sell a stake of
$\alpha = \frac{k(1+r_c)}{\tilde{P}}\%$ to investors. Though investing maximizes social surplus, the original shareholders will not
solicit financing if $\alpha$ is too high. That is, they may prefer to be the sole owners of a less-
valuable firm than partial owners of a more-valuable one. The good shareholders will seek
financing as long as

$$(1 - \alpha)[V_g + (1 + m_g)k] > V_g,$$

where the original shareholders’ terminal wealth if they make the investment is on the left-
hand side. Solving for $\alpha$ yields

$$\alpha < \frac{k(1+m_g)}{V_g + k(1+m_g)} = \alpha_g.$$  \hspace{\textwidth}

(5)

If $\alpha > \alpha_g$, then the losses suffered by shareholders through the dilution of the original assets
exceeds their share of the new investment profits.

A bad firm may invest even though it is never socially optimal if the share $\alpha$ is low
enough. Specifically, bad shareholders will solicit investment if

$$(1 - \alpha)[V_b + (1 + m_b)k] > V_b.$$

Solving for $\alpha$ yields the following condition:

$$\alpha < \frac{k(1+m_b)}{V_b + k(1+m_b)} = \alpha_b.$$  \hspace{\textwidth}

(7)

The relative sizes of $\alpha_g$ and $\alpha_b$ play an important role in the model, illustrated by the
following observation.
Observation 1 If, \( \frac{V_g}{V_b} < \frac{1+m_g}{1+m_b} \), then all good firms receive financing and no bad firms receive financing.

If the maximum stake acceptable to good firms is not acceptable to bad firms, then equity holders offer this stake to all firms and only good firms accept. Such screening is possible if \( \alpha_g > \alpha_b \), which is equivalent to the condition \( \frac{V_g}{V_b} < \frac{1+m_g}{1+m_b} \). For the rest of this section, I assume that this screening condition does not hold--any offer acceptable to good firms is also acceptable to bad firms. The interpretation of the condition is that bad firms are not too much worse than good ones. The difference in returns on the new project (\( \frac{1+m_g}{1+m_b} \)) is less than on the former projects that implicitly determine \( V_g \) and \( V_b \). If the bad firm were worse, the current project returns would be worse than the earlier project returns.

Even if investors cannot make an offer that automatically screens out bad firms, they may be willing to extend financing to all firms without an additional signal if the inherent information asymmetry is not too severe.

Proposition 3 If \( g \geq g^\emptyset_e = \frac{V_g + r_e[V_g + k(1+m_g)] - (1+m_g)(k + V_b)}{(1+m_g)[k(m_g-m_b)+V_g-V_b]} \), then all firms receive financing.

The expected terminal value in this case is the weighted average of the good and bad firm terminal values:

\[
T^\emptyset = g[V_g + k(1 + m_g)] + (1 - g)[V_b + k(1 + m_b)],
\]

implying that equity investors require \( \alpha^\emptyset = \frac{k(1+r)}{T^\emptyset} \). The condition \( \frac{V_g}{V_b} > \frac{1+m_g}{1+m_b} \) implies that \( \alpha_b > \alpha_g \). Both types of firms, therefore, will seek financing as long as \( \alpha^\emptyset \geq \alpha_g \). Setting \( \alpha^\emptyset = \alpha_g \) and solving for \( g \) yields the threshold probability in the proposition. QED.

Investors fail to obtain their required return for bad firms that are financed. They recoup this shortfall from good firms. When the proportion of bad firms is low, the implicit subsidy
from good firms is also low, so good firms accept the financing terms. If \( g < g^0 \), however, the per capita subsidy from good firms is too high. Information asymmetry effectively shuts down financing of all firms. A signal alleviating this asymmetry is necessary to restore financing to some firms.

Note that \( g^0 > g^+ \). That is, the threshold \( g \) above which investors finance all firms without a signal is higher if there is private information. In the absence of private information, the financing terms must provide incentive to the average firm to invest. The dilution of shares acceptable to an average firm, however, may not be acceptable to a firm that knows it is good. Hence, the good firms refuse the terms and no financing occurs. A no-signal equilibrium exists only if the proportion of good firms is higher, lowering the per capita implicit subsidy on good firms. For \( g^+ < g < g^0 \), then, projects with an ex ante positive expected value do not receive financing in the private information, no-signal setting. In the no private information, no-signal setting, all ex ante NPV-positive projects receive financing. This region of financing market failure foreshadows later results.

I now discuss the trading equilibrium with the signal. After the signal realization, there are four types of firms: high-signal good firms, low-signal good firms, high-signal bad firms, and low-signal bad firms. Each type decides whether or not to solicit financing and invest. The investors set signal-contingent stakes \( \alpha_H \) and \( \alpha_L \), respectively. The requirements for the equilibrium are:

1. The strategy of each type of firm is optimal given the strategies of the other types of firms and \( \alpha_H \) and \( \alpha_L \) set by investors.

2. The stakes \( \alpha_H \) and \( \alpha_L \) satisfy Bayes’ Rule, i.e., are consistent with the underlying information structure and the firm strategies.

The following proposition characterizes the equity financing equilibrium:

**Proposition 4** Assume there is private information and that \( \frac{V_g}{V_b} > \frac{k(1+m_a)}{k(1+m_b)} \). Then:
i. If \( g < g^+ \), all high-signal firms receive financing if the underlying classification accuracy is high enough (\( q \geq q^p \)) or if the accounting is conservative enough (\( c \geq c^p \)). Otherwise, high-signal firms do not receive financing. Low-signal firms receive no financing. The expected surplus is weakly increasing in conservatism.\(^5\)

ii. If \( g^+ \leq g < g^p \), all high-signal firms receive financing if the underlying classification accuracy is high enough (\( q \geq q^p \)) or if the accounting is conservative enough (\( c \geq c^p \)). Otherwise, high-signal firms do not receive financing. Low-signal firms receive no financing. The expected surplus is non-monotonic in conservatism.

iii. If \( g \geq g^p \), all high-signal firms receive financing. If conservatism is sufficiently high (\( c \geq \bar{c}^p \)), low-signal firms also receive financing. Minimally conservative accounting (\( c = q \)) maximizes the surplus.

The structure of Proposition 4 is similar to Proposition 2, but there are important differences related to conservatism. First, the quality and conservatism thresholds, \( q^p \) and \( c^p \) are higher than their counterparts \( q^{np} \) and \( c^{np} \) as long as \( \frac{V_g}{V_b} \geq \frac{1+\eta_s}{1+\eta_b} \), the assumption maintained in this section. Second, a relatively high \( g \ (g > g^+ \) is not a sufficient condition for a financing equilibrium to exist, as in the no private information case. To guarantee an equilibrium, the classification errors must also be infrequent enough or the accounting system conservative enough. These differences arise because the equity offer must satisfy the good firm’s incentive compatibility constraint. The expected rate of return given a high signal may exceed the required rate of return, implying that investors would like to provide financing to all firms receiving high signals. If too many bad firms receive high signals, however, the financing terms are relatively dilutive and good firms prefer standing pat. The equilibrium unravels without the participation of the good firms. Either higher quality accounting (higher \( q \)) or more conservative accounting (higher \( c \)) reduce the conditional probability that a high signal is attributable to a bad firm. Thus, the existence of private information expands the range

\(^5\)Please see the appendix for closed form expressions for \( c^p, q^p, g^p \) and \( \bar{c}^p \).
for which conservative accounting is useful to include projects that are \textit{ex ante} viable, in contrast to earlier studies such as Gigler et al (2009), Smith (2007), and Gox and Wagenhofer (2009).

The expected surplus is weakly increasing in conservatism if \( g < g^+ \), for reasons similar to the no private information setting. If no financing equilibrium exists, surplus is 0. Once conservatism is high enough that high firms receive financing, surplus is strictly increasing in conservatism. The comparative statics of the expected surplus are different for more valuable (higher \( g \)) \textit{ex ante} projects, however. The possibility that there does not exist a trading equilibrium even if \( g \geq g^+ \) is the cause. If no financing occurs, the surplus is 0. There is a discrete increase in surplus once conservatism reaches \( c^p_e \) and financing of high-signal firms begins. Further increases in conservatism reduce the expected surplus. That is, a minimum level of conservatism is necessary to generate surplus, but conservatism beyond this level reduces surplus. Hence, the expected surplus is non-monotonic in conservatism.

Figure 2, Panel B illustrates Proposition 4. For the lowest proportion of good firms \((g < g^+)\), investors do not finance high-signal firms unless conservatism is sufficiently high (the point labeled \( c^3 \) in the figure).\footnote{Both \( c^2 \) and \( c^3 \) in Panel B are representations of \( c^i_p \) for different levels of \( g \). In order to avoid confusion by labeling both points \( c^i_p \), I use the other labels.} Beyond this level of conservatism, surplus is weakly increasing in \( c \). For the middle tranche \((g^+ \leq g \leq g^p_e)\), there is no financing unless the accounting is sufficiently conservative, implying, as in the low \( g \) case, a surplus of 0. There is a discrete increase in surplus at the point labeled \( c^2 \), after which surplus is decreasing in conservatism. For the highest proportion of good firms \( g \geq g^p_e \), surplus is decreasing in conservatism until \( c^p_e \). At this point, there is a discrete increase in surplus as low-signal firms (with expected positive values) receive financing. Surplus is fixed with respect to conservatism beyond that because all firms are financed. The fixed level, however, is lower than the surplus for minimally conservative accounting \((c = q)\).

Comparing Panels A and B also illustrates the role that private information plays in the model. For the lowest level of \( g \), the conservatism level for which financing begins is higher for private information. For middle levels of \( g \), high-signal firms receive financing for
minimally conservative accounting if there is no private information. The requirement that financing charges must be low enough to induce privately-informed high-signal good firms to participate results in a higher-than-minimum level of conservatism before surplus exceeds 0. Finally, for the highest level of $g$, the level of conservatism for which low-signal firms receive financing is higher under private information. As a result, there is a discrete increase in surplus at $c = c^p$, albeit to a level below that obtained by minimally conservative accounting (which denies financing to low-signal firms).

An interesting feature of Proposition 4 is that bad firms may receive higher returns than good firms. Good and bad are defined in terms of the return from the new investment, not in terms of final outcomes. If $\frac{V_g}{V_b} \geq \frac{1 + m_g}{1 + m_b}$, it is more costly for good firms to implement the desired action (invest if signal is high) than it is for bad firms. Both types of firms in the high-signal pool sell the same stake (in percentage terms) of the existing assets. Because $V_g > V_b$, this is more costly for good firms. The good firm gets a higher return from the investment, but if the previous condition holds, the incremental return is not enough to offset the extra loss of the existing assets. The signaling equilibrium in Observation 1 is possible only because it is less costly for good firms to agree to the investor’s terms than for bad firms. Note that bad firms may be less likely to be financed in the pooling equilibrium than good firms, especially if $c^p$ is close to 1. Thus, though a financed bad firm outperforms a financed good firm, good firms may have a higher expected ex ante return.

3 Debt financing

The modeling for debt financing is similar to equity financing. I assume that there are competitive financial institutions that expect a rate of return of $r_d$ on all loans. As in the previous section, there are good and bad firms. Both types of firms require financing of $k$ for a project. The cash flow from a good project is $k(1 + m_g)$ with certainty. A bad project yields 0 with probability $d$ and $\frac{k(1 + m_b)}{1 - d}$ with probability $1 - d$, where $0 < d < 1$ is the probability that the bad firm defaults on the loan. I impose this simple structure on the cash flows from good and bad projects in order to facilitate the exposition of the model. Qualitatively similar
results would hold in a model in which both firms can experience a partial or full default. A
good project yields an expected cash flow of \( k(1 + m_g) \) and a bad project yields an expected
cash flow of \( k(1 + m_b) \), with with \( m_g > r_d > m_b \). The bad project, therefore, has a negative
expected value.

In the first-best setting, the good firm borrows \( k \) and the debt service payment is \( k(1 + r_d) \),
yielding a net profit of \( k(m_g - r_d) \). The bad firm repays the bank with probability \( 1 - d \). The
debt service payment necessary to yield return of \( r_d \) is \( \frac{k(1+r_d)}{1-d} \). If the bad firm sought financing
for the investment, the net profit would be \( k(1 + m_b) - (1 - d) \frac{k(1+r_d)}{1-d} = k(m_b - r_d) < 0 \). In
the first-best setting, therefore, the bad firm does not seek debt financing.

The results for debt financing parallel those for equity financing. I first examine the no
private information case. Next, I introduce private information and derive the conditions
necessary for a no-signal financing equilibrium to exist. Finally, I analyze the full model with
private information and the accounting signal. Let \( g_{np} = \frac{r_d - m_b}{m_g - m_b - q(m_g - r_d)} \).

**Proposition 5** Assume there is no private information. Then:

i. The first-best scenario in which all and only good firms receive financing cannot be
   achieved.

ii. If \( g < g^+ \), all high-signal firms receive financing if the baseline classification accuracy is
    high enough \( (q \geq q_{np} = \frac{(r_d - m_b) - g(m_g - m_b)}{(1-g)(r_d - m_b)}) \) or if the accounting is conservative enough
    \( (c \geq c_{np} = \frac{(r_d + q - g) - (1-q) m_b - g(1+q) m_g}{r_d - (1-g)m_b - g m_g}) \), and the expected surplus is weakly increasing in
    conservatism. Low-signal firms do not receive financing.

iii. If \( g \geq g^+ \), all high-signal firms receive financing and the expected surplus is maximized
    by the least conservative accounting \( c = q \).

iv. If \( g \geq g_{np}^+ \) and the accounting is conservative enough \( (c \geq c_{np}^+ = \frac{g q(m_g - r_d)}{g(m_g - m_b) + m_b - r_d}) \),
    then low-signal firms also receive financing.
As in the equity case, low *ex ante* value \( g < g^+ \) projects are financed only if the accounting signal is conservative enough or the baseline classification accuracy is high enough. The surplus is increasing in conservatism for low values of \( g \). Furthermore, only high-signal firms receive financing. The existence of a financing equilibrium does not depend on conservatism if the *ex ante* expected value of the project is positive \( g > g^+ \). The surplus is decreasing in conservatism for this range of \( g \). Finally, conservatism leads to the financing of low-signal firms if the proportion of good firms is high enough. In this case, all projects are financed, including negative expected value bad firm projects. Minimally conservative accounting \( (c = q) \) denies financing to proportionally more bad firms than good firms and therefore maximizes surplus.

In the second-best, private information setting, the good firm profitably borrows for generic debt service payment \( D \) as long as \( D < (1 + m_g)k = D_g \). Similarly, the bad firm profitably borrows as long as \( D < \frac{(1 + m_b)k}{1 - d} = D_b \). If the second-best equilibrium involves pooling of high signal firms, the good firm, because it always repays the loan, faces a higher effective interest rate than the bad firm, which defaults with probability \( d \). In a pooling equilibrium, the investors anticipate that default will occur with probability \( (1 - p_{gh})d \), where \( p_{gh} \) is the conditional probability that a high signal firm is good. The required debt service payment, then, is \( \frac{k(1+r_d)}{1 - (1 - p_{gh})d} \). The good firm always pays this. The bad firm pays the debt service payment with probability \( 1 - d \), for an expected debt service payment of \( (1 - d) \frac{k(1+r_d)}{1 - (1 - p_{gh})d} \). Hence, just as in the equity financing case, pooling can lower the cost of financing enough to make the investment profitable for the bad firm. I begin with a condition on the default rate necessary for achieving the first-best outcome.

**Observation 2** If \( d \leq \frac{m_g - m_b}{1 + m_g} \), then all good firms receive financing and no bad firms receive financing.

The condition implies that \( D_g > D_b \). That is, the maximum debt service payment incentive compatible for good firms exceeds the maximum debt service payment incentive
compatible for bad firms. The credit investor, then, can set $D = D_g$ and only good firms will accept the financing terms, replicating first-best.

I assume that $d > \frac{m_g - m_b}{1 + m_g}$ for the rest of this section. The assumption implies that the bad firm payoff, conditional on success, is higher than the certain good firm payoff of $k(1 + m_g)$. As in the equity case, the economic substance of the assumption is that the bad firm is not too much worse than the good firm. Though the expected return is higher for a good project, the bad project has the higher conditional cash flow. For a worse bad project, both the expected and conditional cash flows would be lower. As in the equity case, creditors are willing to lend to all firms even in the absence of a signal if the proportion of good firms is sufficiently high.

**Proposition 6** If $g > g_d^b = 1 - \frac{m_g - m_d}{d(1 + m_g)}$, then all firms receive financing if there is no signal.

The requirements for the existence of a financing equilibrium are similar to the equity financing section. The bank sets contingent debt service payments $D_H$ and $D_L$. In equilibrium, the strategies of the high and low firms must be optimal given the strategies of the other firms and the debt service payments. Also, the debt service payments must be consistent with the firms’ strategies and Bayes’ Rule.

The next proposition summarizes the debt financing equilibrium.

**Proposition 7** Assume there is private information and that $d > \frac{m_g - m_b}{1 + m_g}$. Then:

i. If $g < g^+$, all high-signal firms receive financing if the baseline classification accuracy is high enough ($q \geq q_d^p = \frac{r_d - m_g + d(1 - g)(1 + m_g)}{(1 - g)(r_d + d - (1 - d)m_g)}$) or if the accounting is conservative enough ($c \geq c_d^p = 1 - \frac{gq(m_g - r_d)}{r_d - m_g + d(1 - g)(1 + m_g)}$). Otherwise, high-signal firms do not receive financing. Low-signal firms receive no financing. The expected surplus is weakly increasing in conservatism.
ii. If \( g^+ \leq g < g_d^p = \frac{r_d d - m_d (1 - d)}{d + q_d + m_d (d - q)} \), all high-signal firms receive financing if the baseline classification accuracy is high enough (\( q \geq q_d^p \)) or if the accounting is conservative enough (\( c \geq c_d^p \)). Otherwise, high-signal firms do not receive financing. Low-signal firms receive no financing. The expected surplus is non-monotonic in conservatism.

iii. If \( g \geq g_d^p \), all high-signal firms receive financing. If conservatism is sufficiently high (\( c > c_d^p = \frac{g_d (m_d - r_d)}{m_d - a (1 - g) (1 + m_d)} \)), low-signal firms also receive financing. Minimally conservative accounting (\( c = q \)) maximizes the surplus.

The proposition is similar to the equivalent equity financing proposition. Sufficiently high baseline classification accuracy (\( q \geq q_d^p \)) induces investors to lower the debt service payment enough that good firms participate in the equilibrium. If the baseline accuracy is not high enough, an additional layer of conservatism (\( c > c_d^p \)) is necessary to alleviate the information asymmetry. The intuition for the effect of conservatism on the expected surplus is the same, also. Even if the proportion of good firms is high, expected surplus may be increasing over a range of conservatism because a minimum level of conservatism is necessary to make a financing equilibrium feasible.

4 Conclusion

This study addresses conservatism in financial reporting. The firm has private information about the expected returns to an investment opportunity. If good project returns are sufficiently higher than bad project returns, investors can set financing terms that are acceptable only to good firms. In this case, the financing results in first-best implementation of projects and maximization of total surplus. If the good and bad project returns are too close to each other, however, investors cannot screen out bad projects in this manner. If the proportion of good firms is high enough, all firms receive financing. Otherwise, no firms do.

In either case, an accounting signal can alleviate the information asymmetry and increase total surplus. The accounting signal has two features. First, there is a baseline classification
error that cannot be purged from the signal due to the underlying complexity of the classification task. Second, an unmodeled regulator can vary the verifiability standard for losses, thereby controlling the level of accounting conservatism. As the standard becomes easier to satisfy, conservatism increases.

I first show that if there is no private information, conservatism is valuable only for projects with a negative \textit{ex ante} expected value. The explanation is that increasing conservatism implicitly sets a higher threshold for project acceptance. When the average project is marginal, careful vetting, achieved by conservatism, is desirable. The opposite is true for positive \textit{ex ante} expected value projects, for which the expected opportunity cost of rejecting good projects is much higher than the expected outlay cost of accepting bad ones. Conservatism results in a deleteriously high rejection rate for good projects, and reduces overall surplus. This result replicates the spirit of earlier results in the literature in Gigler et al (2009) and Smith (2007).

Private information changes the equilibrium. Unless the baseline level of classification error is small, conservatism may be necessary for financing even of high \textit{ex ante} value projects. When firms and investors have the same information, both anticipate a benefit from financing a project if its expected value given the available information is positive. When the signal is present, the desired level of conservatism is that which maximizes total expected profits from investing in high signal projects. This no private information equilibrium, however, may entail good firms having negative realized returns. This would occur under equity financing, for example, if the surrendered stake in valuable existing assets exceeded the new project returns. An informed good firm, therefore, would not participate in the equilibrium, leading to its unraveling. Luring informed good firms back into the equilibrium may require screening out bad firms from the high signal pool through higher conservatism. This is true even if, as is the case for a project with a positive \textit{ex ante} expected value, increasing conservatism is not optimal in the no private information setting. Thus, conservatism may increase the total surplus for either high value or low value projects, expanding its role in the economy beyond that justified by previous theoretical studies.
It is important to note that in the model the accounting signal arrives before the investment. One can interpret the signal as a general signal on the underlying technology of the firm. That is, earnings on prior investments are likely to provide information about current investment opportunities. This is in contrast to Antle, Gigler et al (2009), and Caskey and Hughes (2010), among others, who model a post-investment signal that forms part of a debt covenant determining the allocation of project continuation decision rights between shareholders and creditors. The primary motivation for conservatism in my study is to protect the integrity of the high signal. Good firms that know they are good firms want to be able to convey this information to the markets, and conservatism can help. Though conservatism may reduce the overall informativeness of the reporting system, it always increases the reliability of the high signal, the realization of which is often the necessary, though not sufficient, condition for receiving financing.

The modeling of conservatism in terms of verifiability standards is consistent with the institutional story motivating the empirical proxy for conservatism in Basu (1997). It is impossible to verify analytically that the modeling of conservatism is consistent with the empirical proxy itself for two reasons. First, the proxy is essentially a comparison of the correlation between earnings and returns for positive and negative return samples. A binary model, therefore, does not generate enough data points to perform such a calculation. Second, and more important, there are no earnings in the model. That is, there is no mapping between the underlying signals and balance sheet amounts. This mapping is not necessary to demonstrate the main point of the study: higher conservatism (lower verification standards for losses) can be surplus-maximizing under weaker parameter conditions if firms are privately informed.\(^7\)

Finally, the theory has empirical implications, deriving two industry characteristics that are associated with conservatism. Specifically, one would expect to find conservatism in in-

\(^7\)Because there is no balance sheet, it is also impossible to verify analytically that the modeling is consistent with the proxy for unconditional conservatism in Penman and Zhang (2002). It is easy to devise a mapping that would generate the understatement of assets: write down the asset to market value for a low signal realization and make no adjustment for a high signal realization. I leave it to future research to explore the relations among the underlying information structure, the balance sheet mapping and the empirical proxies.
dustries with a high degree of asymmetric information and in industries with relatively low *ex ante* profitability. A common empirical proxy for asymmetric information is the level of research and development (R&D) expenditures. In this sense, the theory is consistent with the accounting for intellectual property embodied in current United States Generally Accepted Accounting Principles and also in International Financial Reporting Standards. While investment in tangible assets is typically capitalized under both regimes, the threshold for capitalization is higher for intellectual property, especially research and development. Under U.S. GAAP, capitalization occurs only for certain self-generated software assets and for purchased intellectual property. Under IFRS, research costs are typically expensed, but development costs can be capitalized contingent on attaining certain technological and commercial feasibility tests. Determining an empirical proxy for *ex ante* profitability is more difficult. One could argue that projects initiated by R&D intensive firms have a relatively low probability of success (*g* in the model), which is consistent with the optimality of conservatism for low *g* in the model. The profitability thresholds, however, are also functions of the payoffs *m_g* and *m_b*, making it difficult to make a straightforward interpretation. Large sample measures of observed industry profitability and measures of *ex post* returns, are also subject to survivor bias with respect to the estimation of *ex ante* profitability. Thus, caution must be exercised in executing an empirical test of this aspect of the theoretical results.
References


Proofs

Proof of Proposition 1

Proof in text.

Proof of Proposition 2

The expected terminal value of a high-signal firm that seeks financing is

\[ p_{gh}[V_g + k(1 + m_g)] + (1 - p_{gh})[V_b + k(1 + m_b)]. \]

The investor’s payoff if they buy share \( \alpha \) is

\[ \alpha[p_{gh}(V_g + k(1 + m_g)) + (1 - p_{gh})(V_b + k(1 + m_b))]k. \]

Investors must receive the expected return \((1 + r_e)k\), implying that in equilibrium,

\[ \alpha^* = \frac{k(1 + r_e)}{p_{gh}[V_g + k(1 + m_g)] + (1 - p_{gh})[V_b + k(1 + m_b)]}. \]

The expected payoff of a high-signal firm that seeks financing, then, is

\[ (1 - \alpha^*)[p_{gh}[V_g + k(1 + m_g)] + (1 - p_{gh})(V_b + k(1 + m_b))]. \]

The expected payoff of a high-signal firm that does not seek financing is

\[ p_{gh}V_g + (1 - p_{gh})V_b. \]

Solving

\[ p_{gh}V_g + (1 - p_{gh})V_b = (1 - \alpha^*)[p_{gh}[V_g + k(1 + m_g)] + (1 - p_{gh})(V_b + k(1 + m_b))] \]

in terms of \( c \) \((p_{gh} \text{ is a function of } c)\) yields the expression for \( c_{np}^p \) in the proposition. The minimum value of \( c \) is \( q \). Setting \( c_{np}^p = q \) and solving in terms of \( q \) yields the expression for \( q_{np}^p \) in the proposition. Since \( c_{np}^p \) is decreasing in \( q \), \( q_{np}^p \) is the threshold level of accounting quality above which the firm has incentive to finance for all allowable levels of \( c \). Now, \( q \) must be non-negative. Solving \( q_{np}^p = 0 \) in terms of \( g \) yields \( g = \frac{r_e - m_b}{m_g - m_b} = g^+ \). So, if \( g < g^+ \), then the conditions on \( q \) and \( c \) are necessary for financing to occur. If \( g > g^+ \), the high-signal firms
accepts the financing offer for all allowable values of $q$ and $c$. This establishes the existence conditions in parts ii and iii.

The expected terminal value of a low-signal firm that seeks financing is $$p_{gl}[V_g + k(1 + m_g)] + (1 - p_{gl})[V_b + k(1 + m_b)].$$

The investor’s payoff if they buy share $\alpha$ is $$\alpha[p_{gl}(V_g + k(1 + m_g)) + (1 - p_{gl})(V_b + k(1 + m_b))]k.$$

Investors must receive the expected return $(1 + r_e)k$, implying that in equilibrium, $$\alpha^* = \frac{k(1 + r_e)}{p_{gl}[V_g + k(1 + m_g)] + (1 - p_{gl})[V_b + k(1 + m_b)]}.$$

The expected payoff of a low-signal firm that seeks financing, then, is $$(1 - \alpha^*)[p_{gl}(V_g + k(1 + m_g)) + (1 - p_{gl})(V_b + k(1 + m_b))].$$

The expected payoff of a low-signal firm that does not seek financing is $$p_{gl}V_g + (1 - p_{gl})V_b.$$

Solving $$p_{gl}V_g + (1 - p_{gl})V_b = (1 - \alpha^*)[p_{gl}(V_g + k(1 + m_g)) + (1 - p_{gl})(V_b + k(1 + m_b))]$$ yields the condition for $c_{np}^e$ in part iv for the proposition. There cannot be an analogous condition for $q$ because if $c = q$ then low-signal firms are all bad and investors will not finance them. Recalling that $c$ cannot be greater than 1, setting $c_{np}^e = 1$ and solving in terms of $g$ yields the expression for $g_e^p$. That is, if $g < g_e^p$, then the level of conservatism necessary for low signal firms to receive financing exceeds 1, which is impossible. Therefore, $g_e^p$ is the lower bound for $g$ such that low-signal firms receive financing.

If only high-signal firms receive financing, then surplus is $g(1 - c + q)(m_g - r_e) - (1 - g)(1 - c)(r_e - m_b)$. The derivative with respect to $c$ is $-g(m_g - r_e) + g(r_e - m_b)$. Setting this equal to 0 and solving for $g$ yields $g = g^+$. Therefore, in part i, surplus is 0 until $c = c_{np}^e$ and then increasing, and strictly decreasing in part iii.
Proof of Proposition 3

Proof in text.

Proof of Proposition 4

I will first compute $p_{gh}$, the probability that a firm is good given a high signal. 

$$p_{gh} = \frac{g(1-c+q)}{1-c+gq}.$$ 

Let $c^p_c =$ 

$$\frac{(1 + m_g)[k(m_b - gm_b + g(1 + q)m_g) + (1 - g)V_b] + V_g[g(1 + q)m_g + g - 1] - r_e(1 + q)(V_g + k(1 + m_g))}{(1 + m_g)[k((1 - g)m_b + gm_g) - r_e k + (1 - g)V_b] + V_g(g(1 + m_g) - 1 - r_e)}.$$ 

If $c \geq c^p_c$, then the equilibrium strategies are as follows:

1. Good/high firms seek financing and invest.
2. Good/low firms neither seek financing nor invest.
3. Bad/high firm seek financing and invest.
4. Bad/low firms neither seek financing nor invest.

To determine the equilibrium pricing functions, I must first compute the expected terminal value of a firm conditional on investment and a high signal, and the expected required rate of return for a firm conditional on a high signal.

$$\hat{P} = p_{gh}[V_g + (1 + m_g)k] + (1 - p_{gh})[V_b + (1 + m_b)k].$$

Investors set the following required stakes:

1. $\alpha_H = \frac{k(1+\gamma)}{P}$.
2. $\alpha_L = 1$.

I first show that good/high firms maximize the terminal value of the original shareholders’ investment by selling $\alpha_H$% of the shares to investors. I have already shown that good firms are willing to seek financing as long as $\alpha < \frac{k(1+m_g)}{V_g + k(1+m_g)}$. Note that $\alpha_H$ is a function of $c$. Let $c^p_c$ be the degree of conservatism that satisfies:

$$\hat{c} = \alpha < \frac{k(1+m_g)}{V_g + k(1+m_g)}.$$ 


As $c$ increases, the proportion of high signal realizations associated with good firms increases. As a result, $\hat{r}$ decreases and $\hat{P}$ increases, implying that $\alpha_H$ is decreasing in $c$. If the original shareholders are willing to sell shares at $c^p_e$, they are also willing to sell shares for $c > c^p_e$. So the good/high strategy is optimal. The information structure limits $c$ to the $[q, 1]$ interval.

It is possible that $c^p_e < q$. If so the good/high firm invests for all allowable $c$. I define $q^p_e$ as the solution in terms of $q$ to $c^p_e(q) = q$, or

$$\frac{(1 + m_g)[k((1 - g)m + gm_b) - r_e k + (1 - g)V_b] + V_g(g(1 + m_g) - 1 - r_e)}{(1 - g)(1 + m_g)(km_b + V_b) - V_g - r_e(k + km_g + V_g)}.$$ 

Thus, for all $q \geq q^p_e$, $c > \hat{c}$, and the proposition holds. I now address bad/high firms. I have already shown that bad firms are willing to seek financing as long as $\alpha < \frac{k(1 + m_g)}{V_b + k(1 + m_b)}$. Now, $\frac{k(1 + m_g)}{V_b + k(1 + m_b)}$ is greater than $\frac{k(1 + m_g)}{V_g + k(1 + m_g)}$, the threshold $\alpha$ for good firms, as long as $\frac{V_b}{V_g} > \frac{1 + m_a}{1 + m_b}$, which I have assumed. Therefore, the bad/high strategy is optimal. Given $\alpha_L = 1$, neither good nor bad firms with bad signal realizations will sell.

Given that both good and bad firms with high signal realizations seek financing, Bayes’ Rule requires that the equilibrium $\alpha_H$ be an average of the good and bad firms shares weighted by their relative likelihoods. Therefore, $\alpha_H$ in the proposition is computed correctly. Given that neither good nor bad firms receiving low signals seek financing, any $\alpha_L$ forms part of an equilibrium as long as it is too high to induce good firms to seek financing. This proves the part of the proposition for which $c > c^p_e$.

If $c < c^p_e$, then I have already shown that the $\alpha_H$ that pools good and bad firms with high signal realizations is high enough that the good firms do not sell. Bad/high firms would willingly sell at that price, but, given the good/high firms’ abstention, investors will set $\alpha > \alpha_b$. As a result, no financing or investment occurs for any type of firm.

The comparative statics are implied by the no-trade intervals of $c$. Following the proof of Proposition 2, equating the low-signal equilibrium expected firm payoffs with and without financing and solving for $c$ yields

$$c^p_e = \frac{gq(m_g - r_e)[V_g + k(1 + m_g)]}{(1 + m_g)[(1 - g)V_b - r_e k + k((1 - g)m + gm_b) + V_g(g(1 + m_g) - 1 - r_e)].}$$

As long as $c \geq c^p_e$, enough good firms receive low signals that investors are willing to provide financing to low-signal firms. Setting $c^p_e = 1$ and solving for $g$ yields the minimum proportion

$$29$$
of good firms such that this can hold:

\[ g^p_c = \frac{(1 + m_g)[V_b - k(r_e - m_b)] - (1 + r_e)V_g}{(1 + m_g)[V_b + k(m_b - (1 - q)m_g - qr_e)] - V_g[1 + r_e q + (1 - q)m_g]}. \]

**Proof of Proposition 5**

The expected default rate for a high signal is \( p_{gh} \cdot 0 + (1 - p_{gh})d \). The bank payoff as a function of the debt service payment \( D \) is \( D[1 - (1 - p_{gh})d] \). The credit investor requires a return of capital of \( k(1 + r_d) \). Let the debt service payment that solves \( D[1 - (1 - p_{gh})d] = k(1 + r_d) \) be \( D^* \). The expected return for an average firm is

\[ k[1 + p_{gh}m_g + (1 - p_{gh})m_b] - (1 - (1 - p_{gh}d))D^*. \]

The firm will not invest unless it breaks even. Setting the expected return equal to 0 and solving for \( c \) (recall that \( p_{gh} \) is a function of \( c \)) yields \( c^{np} = \frac{r_d(1 + q^{d}) - (1 - q)m_b - g(1 + q)m_g}{r_d - (1 - g)m_b - gm_g} \).

Setting \( c^{np}_d(q) = q \) and solving for \( q \) yields \( q^{np}_d = \frac{(r_d - m_b - g(m_a - m_b)}{(1 - g)(r_d - m_b)} \). The threshold \( q^{np}_d \leq 0 \) if and only if \( g \geq g^+ \). For \( g > g^+ \), therefore, the firm accepts the financing offer for all values of \( c \) and \( q \). The thresholds \( c^{np}_d \) and \( q^{np}_d \) bind only for \( g < g^+ \).

Now the low signal. The expected default rate is \( (1 - p_{gl})d \). The bank payoff as a function of the debt service payment \( D \) is \( D[1 - (1 - p_{gl})d] \). The debt service payment \( D^* \) solves \( D[1 - (1 - p_{gl})d] = k(1 + r_d) \). The expected return for an average firm is

\[ k[1 + p_{gl}m_g + (1 - p_{gl})m_b] - (1 - (1 - p_{gl}d))D^*. \]

The firm will not invest unless it breaks even. Setting the expected return equal to 0 and solving for \( c \) yields \( c^{np}_d = \frac{r_d}{g(m_g - r_d) + m_b - r_d} \). There cannot be an analogous condition for \( q \) because if \( c = q \), only bad firms generate the low signal and financing cannot occur. Solving for the \( g \) such that \( c^{np}_d \) attains its upper bound of 1 yields \( g^{np}_d = \frac{r_d - m_b}{m_g - m_b - q(m_g - r_e)} \). If \( g \) is lower than \( g^{np}_d \), then low-signal firms do not receive financing.

The analysis of the surplus is similar to the equity case. I do not repeat it here.

**Proof of Proposition 6**

The firms have private information, but there is no signal. The bank expects a default rate of \( d(1 - g) \). The debt service payment \( D^* \) solves \( D[1 - (1 - g)d] = k(1 + r_d) \). The good firm’s
payoff is \( k(1 + m_g) - D^* \). It will reject any debt service payment higher than \( k(1 + m_g) \). Setting \( D^* = k(1 + m_g) \) and solving for \( g \) yields \( g_d^0 = 1 - \frac{m_g - r_d}{d(1 + m_g)} \). So, financing occurs if and only if \( g \geq g_d^0 \).

**Proof of Proposition 7**

The bank requires a return of \( k(1 + r) \). If the signal realization is high, the bank receives the debt service payment with probability \( 1 - d(1 - p_{gh}) \). The probability that the firm is bad is \( 1 - p_{gh} \), and bad firms default with probability \( d \). Setting \( k(1 + r_d) = [1 - (1 - p_{gh})d]D \) and solving for \( D \) yields \( D_{eq} = \frac{k(1 + r_d)}{1 - d(1 - p_{gh})} \). The condition \( d > \frac{m_g - m_b}{1 + m_g} \) implies that the bad firm incentive-compatible debt service payment is higher than the good firm incentive-compatible debt service payment. If the good firm chooses to borrow, therefore, the bad firm will also. The good firm incentive compatible debt service payment is \( D_g = k(1 + m_g) \). Setting \( D_{eq} = D_g \) and solving for \( c \) yields \( c_d^g = 1 - \frac{g_d(m_g - r_d)}{r - m_g + d(1 - g)(1 + m_g)} \). If \( c > c_d^g \), then the debt service payment is incentive compatible for both types of firms. Setting \( c_d^p = q \) and solving for \( q \) yields \( q_d^p = \frac{r_d - m_g + d(1 - g)(1 + m_g)}{(1 - g)(r_d + d - (1 - d)m_g)} \). If \( q \geq q_d^p \), then the debt service payment is incentive compatible for both firms. Also, \( c_d^p < q_d^p \), which is that lowest permissible value of \( c \) in the information structure, automatically satisfying that condition.

Now consider the low signal. Setting the debt service payment required for the bank to break even equal to the maximum debt service payment acceptable to a good firm yields the conservatism threshold in the proposition. A threshold greater than 1 violates the parameter restriction, implying that the low signal will not be funded. Setting the threshold equal to 1 and solving in terms of \( g \) yields \( g_d^p \). For \( g \) above this value there will always be a level of conservatism allowing financing of low firms. The relevant thresholds for \( g \) and \( c \) are in the body of the proposition.

The analysis of the surplus is similar to previous propositions and I will not repeat it here.
Glossary of notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>probability that a firm is good</td>
</tr>
<tr>
<td>$V_i$</td>
<td>initial (private) value of firm of type $i$</td>
</tr>
<tr>
<td>$k$</td>
<td>investment</td>
</tr>
<tr>
<td>$m_i$</td>
<td>return to investment for firm of type $i$</td>
</tr>
<tr>
<td>$r_e$</td>
<td>required rate of return for equity investors</td>
</tr>
<tr>
<td>$c$</td>
<td>degree of conservatism</td>
</tr>
<tr>
<td>$q$</td>
<td>baseline level of classification accuracy</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>percentage stake in firm (variety of contexts)</td>
</tr>
<tr>
<td>$r_d$</td>
<td>required rate of return for debt investors</td>
</tr>
<tr>
<td>$d$</td>
<td>probability that a bad firm defaults on debt</td>
</tr>
<tr>
<td>$D$</td>
<td>debt service payment (variety of contexts)</td>
</tr>
<tr>
<td>$p_{gh}$</td>
<td>probability firm is good given high signal</td>
</tr>
<tr>
<td>$p_{gl}$</td>
<td>probability firm is good given low signal</td>
</tr>
</tbody>
</table>
Figure 1 Panel A: Information Structure

```
q,  
   /\   \ 1-c+q  
 Good Firm       High Signal
   \   /  c-q
  1-g       Bad Firm
            \ /  \c
             q
```

Figure 1 Panel B: Maximum Conservatism

```
q,  
   /\   \ q  
 Good Firm       High Signal
   \   /  1-q
  1-g       Bad Firm
            \ /  \1
             q
```

Figure 1 Panel C: Minimum Conservatism

```
q,  
   /\   \ 1  
 Good Firm       High Signal
   \   /  1-q
  1-g       Bad Firm
            \ /  \q
             q
```

Only good firms can generate high signal.

Only bad firms can generate low signal.
Figure 2 Panel A: No Private Information

- High $g$ ($g \geq g_+^{np}$): All firms financed if $c > c_+^{np}$.
- Surplus weakly decreasing in conservatism.
- Surplus maximized at $c = q$.

- Medium $g$ ($g_+^{np} > g > g_+^{e}$): All high signal firms financed.
- Surplus strictly decreasing in conservatism.
- Surplus maximized at $c = q$.

- Low $g$ ($g \leq g^*$): All high signal firms financed if $c > c_3^{np}$.
- Surplus weakly increasing in conservatism.
- Surplus maximized at $c = 1$.

$g$ is the proportion of good firms.

Figure 2 Panel B: Private Information

- High $g$ ($g \geq g_+^{p}$): All firms financed.
- Surplus non-monotonic in conservatism.
- Surplus maximized at $c = q$.

- Medium $g$ ($g_+^{p} > g > g_+^{e}$): All high signal firms financed if $c > c_2^p$.
- Surplus non-monotonic in conservatism.
- Surplus maximized at $c = c_2^p$.

- Low $g$ ($g \leq g^*$): All high signal firms financed if $c > c_3^p$.
- Surplus weakly increasing in conservatism.
- Surplus maximized at $c = 1$.

$g$ is the proportion of good firms.