An Analysis of Advertising Payments in Franchise Contracts

Ram C. Rao
Shuba Srinivasan

ABSTRACT. National advertising is an important ongoing marketing activity in a franchise arrangement. A majority of franchisors require franchisees to pay an advertising royalty as a percentage of gross revenues while some require franchisees to pay a fixed advertising fee. These payments are earmarked for national advertising. We investigate the relationship between the franchisor’s profits and the different types of advertising payments in franchise contracts. Our model incorporates the idea that the franchisor and franchisee are in an ongoing relationship where there is demand uncertainty. We show that specification of an advertising payment in the form of a fixed fee or a royalty is better than no specification since it commits the franchisor to invest the payments in advertising. We demonstrate that the advertising royalty specification is more flexible since it permits the advertising expenditure to be adjusted based on information that is not available at the time the contract is written.

KEYWORDS. Franchise contracts, advertising payments, vertical marketing system

Ram C. Rao is Founders Professor of Marketing, School of Management, The University of Texas at Dallas, Richardson, TX 75083-0688. Shuba Srinivasan is Assistant Professor of Marketing, The A. Gary Anderson Graduate School of Management, University of California, Riverside, CA 92521-0203.

The authors thank the Editor, Lou E. Pelton, and the referees for their comments, which have greatly improved this paper. They are also grateful to Rajiv Lal, Kannan Srinivasan and the seminar participants at the University of Texas at Dallas, the University of California, Riverside and Carnegie Mellon University for their invaluable suggestions.

Journal of Marketing Channels, Vol. 8(3/4) 2001 © 2001 by The Haworth Press, Inc. All rights reserved.
1. INTRODUCTION

Franchising has been in a growth phase in recent years. Business-format franchising accounts for much of this growth. For example, between 1984 and 1994, franchise businesses grew from 440,000 establishments to 663,200 according to the International Franchise Association. In business-format franchising, a franchisor offers not only the brand name and the product/service, but also the entire business format to franchisees (U.S. Department of Commerce, 1987). The franchisor offers help in setting up the outlet, offers its method of doing business and provides ongoing services such as national advertising. The franchisee, in turn, contributes to the system by managing the franchised outlet and providing services at the retail level. Central to business-format franchising is the franchise contract. Many franchise contracts specify a payment by each franchisee to the franchisor that is earmarked for national advertising expenses by the franchisor (see, e.g., Bond, 1994). This payment is in addition to other transfers from the franchisee to the franchisor—a fixed fee, normally paid at the time the parties enter into the contract and a sales royalty that specifies payments as a percentage of realized gross sales by the franchisee (Rubin, 1978). Franchisors typically use one of two methods in collecting advertising payments. Some franchisors ask their franchisees to pay a fixed (lump-sum) amount as the advertising fee. This is similar in form to the franchise fee. Alternatively, franchisors collect a percentage of franchisees’ gross sales as the advertising fee. This is similar to the sales royalty. However, an important difference between advertising payments and the franchise fee/sales royalty is that the franchisee fee and royalty are part of the franchisor’s revenue stream while the franchisor is committed to invest the advertising payments only on national advertising. Table 1a and 1b show the extent to which franchisors specify advertising payments in franchise contracts based on data from the Sourcebook of Franchise Opportunities (Bond, 1995) and Bhattacharya and LaFontaine (1995) respectively.

An inspection of Tables 1a and 1b suggest that a majority of the franchisors specify an advertising royalty rather than a fixed advertising fee. Several important questions arise. Why do most franchisors charge an advertising royalty rather than an advertising fee? How are these two forms of advertising payments different? And, who benefits from these payments? We address these questions by considering a
model of a franchisor with a single franchisee. Our paper offers novel findings in this regard. Since most franchisors do invest in some form of national advertising, in the absence of such payments toward advertising from franchisees, they would have to fund advertising from their revenues received from franchise fees, sales royalties and the mark-up on products sold to the franchisees. Such a method of financing leaves franchisees unsure of how much the franchisor will invest in advertising. As a result, channel profits are not maximized in equilibrium. In contrast, our results demonstrate that a commitment to advertising by the franchisor leads to better channel coordination; this is the explanation we offer for the advertising payment specifications in franchise contracts. Moreover, since advertising payments improve channel
coordination, both the franchisor and the franchisee benefit from such advertising specifications.

A second and more important issue we address in this paper is the manner in which advertising payments are specified in a franchise contract. We compare the relative profitability of using the fixed advertising fee, using the advertising royalty and using no advertising payments. On the one hand, using the sales-based advertising royalty adversely affects the franchisees' choice of price and service. Any level of advertising royalty has some negative effect on the franchisees' optimal price and service decisions while providing incentives to the franchisor (Rubin, 1978; Lal, 1990). On the other hand, the fixed advertising fee allows the franchisor to achieve better coordination on price and service. However, in practice, it is more common to see advertising payments specified as a royalty rather than a fixed fee. Our results show that if all contingencies affecting demand cannot be predicted at the time the parties enter into the contract, the advertising royalty specification is more flexible than the advertising fee specification. Thus, depending on the level of uncertainty, either an advertising fee or an advertising royalty specification may be optimal. Since demand uncertainty is common, advertising royalty specifications are more commonly observed than fee specifications.

1.1. Dual Moral Hazard and Franchising Literature

A number of recent studies are closely related to our work on the elements of franchise contracts. Since we develop a dual moral hazard model of contractual arrangements in this paper, we will summarize the important studies on this topic. The notion that dual moral hazard can explain various institutional arrangements is not new (Milgrom & Roberts, 1992). Indeed, Rubin (1978) first suggested it as an explanation for franchising. Eswaran and Kotwal (1985) and Dybvig and Lutz (1993) have formalized these arguments in the contexts of sharecropping and product warranties respectively.

The dual moral hazard model arises from the assumption that both the franchisor and the franchisee undertake privately observed actions that affect the demand for the product. By reducing the marketing effort, the franchisor can save costs, and the decrease in marketing effort can be observed only indirectly through the decrease in demand. Similarly, the franchisee can obtain direct benefits at the expense of decreasing demand by reducing the level of service. Introducing a
royalty payment has opposing impacts on the two moral hazard problems: having a royalty increases the franchisor’s incentives for marketing effort but decreases the franchisees’ service incentives. The optimal royalty in the second-best solution represents a trade-off between these two moral hazard incentives.

In the context of franchising, Mathewson and Winter (1985) examine the factors affecting the elements of franchise contracts using principal-agent literature. They predict that new franchise chains with uncertain brands names should have lower franchise fees, lower advertising and less monitoring than more established chains. Lal (1990), addressing royalty payments in franchise contracts, shows that they balance the incentives of the franchisor and the franchisee to provide on-going marketing effort and service respectively. This is consistent with the evidence in Agrawal and Lal (1992), LaFontaine (1992), Sen (1993) and Shane (1998). LaFontaine (1992) presents empirical evidence in support of two-sided hidden action models. She finds that the royalty rate is positively related to the number of outlets using the franchisor’s brand name and the projected future growth in number of outlets using the franchisor’s brand name.

More recently, Desai (1997) studied the role of advertising fee in business-format franchising. He considers the case of a single franchisor dealing with two identical franchisees and finds that the advertising fee allows the franchisor to commit to a specific level of advertising spending and hence is better than no advertising payments. Further, he argues that when franchisees’ markets differ in how advertising affects sales, the franchisor may prefer a sales-based advertising fee. In a departure from Desai (1997), our model specifically incorporates the idea that the franchisor and the franchisees are in a long-term relationship with ongoing demand uncertainty (see, for e.g., Villas-Boas, 1998). Given this environment, there are significant costs to re-specify the elements of the contract as demand conditions change. Under these conditions, we show that advertising royalty provisions in a franchise contract are more likely when contingencies affecting demand cannot be predicted at the time the parties enter into the contract.

The rest of the paper is organized as follows. In Section 2, we formulate a model of franchising as a two-stage game with the first stage corresponding to the design of the franchise contract and the second stage corresponding to the marketing decisions based on information not available at the time the contract is written. Through a
numerical example, we demonstrate the main results of our paper. In Section 3, we undertake a formal analysis of our model and establish the results in a more general way. In Section 4, we provide our conclusions.

2. MODEL OF THE FRANCHISOR-FRANCHISEE INTERACTION

We conceptualize the franchisor-franchisee interaction as a leader-follower game following past research (McGuire & Staelin, 1983; Lal, 1990). The franchisor is the leader, and the franchisee is the follower. The leader announces some of her decisions first, in Stage I, and the follower and the leader take actions subsequently in Stage II. Thus, the franchisor is a Stackleberg leader with respect to her choices in the first stage but not in the second stage. The second stage choices of the franchisor and the franchisee are assumed to be made simultaneously and correspond to a Nash equilibrium in the second stage of the game. Thus, our game structure is similar to that of Lal (1990).

Our model focuses on a single franchisee-franchisor pair. Two other possibilities include a single franchisor-multiple franchisees model and a single franchisor-multiple franchisees-multiple franchisor-owned units model. While the latter two models would better reflect actual franchise operations, the exposition is clearer with the former model. Further, from a substantive perspective, we expect the results to be similar, and we leave for future research further extensions of the proposed model.

In an important departure from Lal (1990), we view the second stage as conditional on new information not available at the time the contract is written. This new information pertains to the knowledge in an expected sense of the state of demand. Discussing long-term contracts executed under conditions of uncertainty, Williamson (1979) observes: “Problems of several kinds arise. First, not all future contingencies for which adaptations are required can be anticipated at the outset. Second, the appropriate adaptations will not be evident for many contingencies until the circumstances materialize.” In this sense, our conceptualization is consistent with Villas-Boas (1998) and involves the choice of terms of the contract by the franchisor and marketing actions by both the franchisor and the franchisee. Moreover, these marketing actions are not fully specified in the contract. In our model, Stage I corresponds to the choice of the terms of the
contract and Stage II to the marketing actions. The new information in our model refers to the demand being in one of two states: high or low.

2.1. Game Structure

In stage I, the franchisor chooses two elements of the contract: the royalty rate and the fixed fee.\(^2\) Depending on the contract, the franchisor may also specify an annual advertising fee or an advertising royalty.\(^3\) At the time these are chosen, the franchisor knows that demand can be in one of two states--high or low--with a probability of \(\pi_1\) and \(\pi_2\), respectively, where \(\pi_1 + \pi_2 = 1\). In the second stage, assuming the franchisee accepts the contract, each of the two parties undertakes actions in its best interest while observing the terms of the contract. These actions include the choice of price and service by the franchisee, quality and possibly advertising by the franchisor. Consider the case of a fast food outlet where the service level refers to aspects such as the hours of operation of the outlet and the friendliness of service; quality includes control over aspects such as product assortment, ingredients and recipes for new products. These aspects of quality and service are chosen on an ongoing basis. In addition, the franchisor has control over the advertising strategies at the national and regional level. Next, we state the assumptions of our models.

2.2. Assumptions

Assumption 1: The demand \(D\) for the product is assumed to be given by:

\[
D_i = \phi_i \cdot \psi_i - \theta_i \cdot \alpha \cdot \psi_i - \delta \cdot \xi \cdot \psi_i, \quad i = 1, 2
\]

Where \(\phi_i, \psi_i, \theta_i, \alpha, \delta, \xi\) are \(\geq 0\), and \(i = 1, 2\). In the demand equation, \(D_i\) is the demand with \(i = 1\) corresponding to the high demand state and \(i = 2\) corresponding to the low demand state. The exogenous random shock influences demand, is assumed to have a mean of zero and is identically distributed for both high and low demand states. The advertising, quality, service and price sensitivity parameters are \(\phi, \psi, \theta, \alpha\) and \(\delta, \xi\), respectively. The probability of occurrence of the high demand state is \(\pi_1\), and the probability of occurrence of the low demand state is \(\pi_2\); \(p_i\) is the price to the consumer; \(s_i\) is the level of service; \(q_i\) is the quality.
level; and \( A_i \) is the level of advertising. The assumption implies that a higher level of marketing effort, whether it is service chosen by the franchisee or quality and advertising chosen by the franchisor, is productive. We have used the linear specification for the demand function following prior work (e.g., McGuire & Staelin, 1983; Lal, 1990) for tractability. The intercept \( \beta_1 \) corresponds to the high demand state, and \( \beta_2 \) corresponds to the low demand state. In the rest of the paper, we will choose \( \beta_2 = 1 \) and \( \beta_1 = 1 \) unless otherwise specified. This makes the exposition simpler without loss of generality.

Assumption 2: The cost function is as follows. The franchisees’ cost, \( C(s) \), so providing service at level \( s \) is assumed to be given by

\[
C(s) = \frac{q^2}{2}
\]

The franchisor’s cost, \( C(q) \), of providing a quality level of \( q \) and an advertising level of \( A \) are assumed to be given respectively by

\[
C(q) = \frac{q^2}{2}
\]

and

\[
C(A) = \frac{A^2}{2}
\]

We make this assumption to capture diminishing returns to effort. In other words, the cost incurred in effort in the form of quality and advertising chosen by the franchisor and service chosen by the franchisee increases with effort at an increasing rate.4

Assumption 3: Both the franchisor and the franchisee are risk neutral (i.e., they maximize expected profits).

Assumption 4: The minimum net profits that the franchisee expects are assumed to be \( U \). Without the loss of generality, we let \( U = 0 \).

2.3. Advertising Payment Specifications and Profit Maximization

At the beginning of the game, the franchisor declares the fixed fee \( f \), the royalty rate \( r \) and possibly an advertising payment. First, the adver-
Advertising payment may be specified as an advertising fee \( C(A) \) (denoted by program \( P1 \)). Second, the advertising payment may be specified as an advertising royalty \( a \) (denoted by program \( P2 \)). Finally, the advertising may be left unspecified (denoted by program \( P3 \)). The franchisor can spend the advertising payments collected only on advertising as noted earlier. However, most franchise contracts also explicitly note that there is no guarantee that the regional allocation of advertising will match the in-take distribution. We do not account for this externality issue since typically the advertising payments go towards national rather than regional advertising as pointed out by Raab and Matusky (1987) and Townsend and O’Leary (1987). A future extension of this model could refine \( P2/P1 \) to account for this externality issue.\(^5\)

Depending on whether or not the franchisor specifies advertising in the contract, and if so, how, there would be a distinct choice of retail price and service in the second stage by the franchisee. In addition to the first stage choices, the franchisor also chooses \( q \), the level of quality in the second stage simultaneously with the franchisee’s choice of retail price \( p \) and service level \( s \).

As noted earlier, there are three possible methods of specifying advertising payments in franchise contracts. The first method is to specify an advertising fee \( C(A) \) in the contract; that is, advertising expenditure is a first stage choice. Of course, the fee \( C(A) \) uniquely determines the advertising level \( A \). Note that in this case, the level of advertising \( A \) and the corresponding expenditure \( C(A) \) do not depend on information in the second stage. The second method is to specify an advertising royalty \( a \) in the contract, again a first stage decision. The actual advertising level \( A_i \) and the expenditure \( C(A_i) \) will depend on other second stage decisions and therefore on the information available only in the second stage. The third method is to leave advertising unspecified in the contract. In this case, the advertising level \( A_i \), and equivalently, the expenditure \( C(A_i) \), is a second stage decision. As a result, the franchisor’s decision depends on information in the second stage. For each of these cases, the franchisor’s optimization problem in the first stage and second stage differs. We will treat the case in which \( C(A) \) is specified in the contract as a base case. In the base case, the franchisor faces the following program denoted as \( P1 \).
Program P1

\[
\begin{align*}
\max_{\pi} & \quad \sum_{i=1}^{n} \mu_i \pi_i \cdot D_i \cdot C_i(A_i) \cdot j \cdot C_i(A_i) \\
\text{subject to} & \quad \sum_{i=1}^{n} \pi_i \cdot D_i \cdot C_i(A_i) \cdot j \cdot C_i(A_i) = 1.2 \\
& \quad \alpha_i \cdot \pi_i \cdot D_i \cdot C_i(A_i) \cdot j \cdot C_i(A_i) = 1.2 \\
& \quad \sum_{i=1}^{n} \pi_i \cdot D_i \cdot C_i(A_i) \cdot j \cdot C_i(A_i) = 0 \\
& \quad 0 < f < \infty \\
& \quad f \geq \varepsilon \\
& \quad C_i(A_i) \geq c
\end{align*}
\]

The conditions 6 and 8 represent the incentive compatibility (IC) constraint and the individual rationality (IR) constraint of the franchisee. The IC constraint captures the idea that the franchisor anticipates the franchisee’s optimal actions in the second stage. The IR constraint captures the minimum utility that the franchisee seeks. It guarantees that the franchisee’s minimum utility requirement is met in an expected sense. The IR constraint for the franchisee’s expected profits is based on the assumption that he is risk-neutral (from Assumption 3). Condition 7 states that the franchisor will choose quality level \( q \) in the second stage so as to maximize her profits given the information in the second stage.

We now turn to the contract that specifies an advertising royalty. The franchisor specifies \( a \), the advertising royalty and \( r \), the sales royalty. The actual level of advertising, \( A_i \), will depend on the second stage decisions and hence on the information available in the second stage. When the advertising royalty is specified in the contract, the program is denoted by P2.
As in Program P1, the conditions 13 and 16 represent the incentive compatibility (IC) constraint and the individual rationality (IR) constraint of the franchisee. As before, Condition 14 states that the franchisor will choose quality level $q_i$ optimally in the second stage. Program P2 differs from P1 in two ways. First, the franchisor chooses advertising royalty $a$ as opposed to advertising fee $C(A)$, as reflected in 12. Second, Constraint 15 captures the relationship between the contracted advertising royalty $a$ and the actual advertising expenditure $C(A_i)$. Our formulation is consistent with closure under rational expectations. It implies that the money spent on advertising is equal to the advertising payments received in the corresponding demand state. This condition is also evident in the language commonly used in the franchise documents. For example, Hardee’s 1996 Uniform Franchise Offering Circular states, “All contributions to Hardee’s National Advertising Fund (HNAF), and any earnings on contributions, are used
exclusively to meet costs of maintaining, administering, conducting and preparing advertising, marketing, public relations and/or promotional programs and materials and any other activities which we believe will enhance the image of the Plan” (p. 31).

Finally, we turn to the case where no advertising payment is specified in the contract. The franchisor’s choice of the level of advertising in the second stage, \(A_i\), will depend on the information available in that stage. The franchisor solves Program P3 as follows.

**Program P3**

\[
\text{\text{\begin{align*}
\text{max} & \quad \sum_{i=1}^{n} \pi_i y_i (q_i) - C(q_i) - f \\
\text{subject to} & \quad (19) \\
\text{subject to} & \quad \text{Equations 20, 21 and 23 represent the incentive compatibility (IC)} \\
\text{subject to} & \quad \text{constraint of the franchisee, the franchisor’s choice of quality level } q \\
\text{subject to} & \quad \text{in the second stage and the individual rationality (IR) constraint of the} \\
\text{subject to} & \quad \text{franchisee. Program P3 differs from Programs P1 and P2 in two ways.} \\
\text{subject to} & \quad \text{First, the franchisor chooses only the sales royalty } r \\
\text{subject to} & \quad \text{and the franchise fee } f \text{ as reflected in the objective function 19. Second, equation 22} \\
\text{subject to} & \quad \text{states that the franchisor will choose advertising level } A_i \\
\text{subject to} & \quad \text{in the second stage based on information available in the second stage.} 
\end{align*}}
\]

Equations 20, 21 and 23 represent the incentive compatibility (IC) constraint of the franchisee, the franchisor’s choice of quality level \(q\) in the second stage and the individual rationality (IR) constraint of the franchisee. Program P3 differs from Programs P1 and P2 in two ways. First, the franchisor chooses only the sales royalty \(r\) and the franchise fee \(f\) as reflected in the objective function 19. Second, equation 22 states that the franchisor will choose advertising level \(A_i\) in the second stage based on information available in the second stage.
Before we solve for the three equilibrium contracts corresponding to each of these programs, it is useful to explore a numerical example.

2.4. Numerical Example

The first set of questions to ask is: Which of these three contracts, i.e., the method of specifying or choosing advertising is the most desirable? Is there one that is always superior? A second set of questions is: What is the effect of the probability of occurrence of the high demand state or low demand state on the method of choosing advertising? How does advertising expenditure depend on the form of the contract? And, what is the effect of the probability of occurrence of the high demand state or low demand state on the advertising expenditure? We provide answers to these questions below.

For the numerical example, let the intercepts of the two states of demand $a_1 = 2$ and $a_2 = 1$; the price sensitivity parameter $= 2.3$; and the advertising sensitivity parameter $= 1.1$.

The difference in profits between programs P1 and P3 as a function of $\lambda$, the probability of occurrence of the high demand state, is shown in Figure 1a. In Figure 1b, we illustrate the difference in profit between Programs P2 and P3 as a function of $\lambda$.

Note that the desirability of each contractual offer depends on the uncertainty captured through $\lambda$. From Figure 1a, we can see that a
contractual offer with no specification is not optimal under any level of uncertainty. The franchisor can do better by specifying advertising in the form of an advertising fee. When the franchisor uses the advertising fee, it can commit to the coordinated advertising level and induce the franchisees to choose the coordinated price and service levels. Therefore, by using the fixed advertising fee, the franchisor can achieve coordination on price, service and advertising. The channel profits are therefore higher with an advertising specification than with no advertising specification. Mathewson and Winter (1985) and Desai (1987) also obtain a similar result.

The franchisor can also use the advertising royalty for advertising commitment. Given that leaving advertising unspecified is not desirable, the question is: Is it better to specify advertising in the form of an advertising fee or in the form of an advertising royalty? This depends on the need to adjust advertising based on the second stage information. In Figure 1b, we see that the specification of advertising in the form of an advertising royalty is optimal for \( \ell \leq h \) while specification in the form of an advertising fee is optimal for \( \ell < \ell \) and \( h > h \). The values of \( \ell \) and \( h \) clearly will depend on \( \ell \) and \( h \), the intercepts corresponding to the high and low demand states respectively. The intuitive explanation for this is as follows: The advertising fee specification implies that the franchisor would have to commit to a level of advertising which is the same under both the high and low
demand states. This is a particular manifestation of the more general issue of commitment as noted by Hadfield (1990): “... the franchisor would have to negotiate today over how best to respond to long-term vagaries of the automobile industry or the fast-food market, or the demand for tax services ...” Clearly, a commitment to the level of advertising is not flexible under different demand states. In contrast, the advertising royalty specification is a more flexible instrument than a single fixed advertising fee and takes into account the information on the demand state in the second stage. When \( \gamma \) is close to zero or one, the need for flexibility is less than when \( \gamma \) is intermediate. However, any level of advertising royalty has some negative effect on the franchisees’ optimal price and service decisions. In other words, the service and price decisions are not at the coordinated levels when an advertising royalty is specified. Hence, the trade-off is between the flexibility of the advertising royalty and the adverse consequences of the advertising royalty on the franchisees’ choices of price and service. When a fixed advertising fee is specified, it induces the franchisee to make better price and service decisions.

Let us next turn our attention to the effect of the probability of occurrence of the high demand state or low demand state on the advertising payment which is either the advertising fee \( C(A) \) or the advertising royalty \( a \). We denote the optimal decision by the superscript * and the decision corresponding to the case \( \gamma = 0 \) by the superscript 0. Thus, we wish to compare \( C(A^*) \) with \( C(A^0) \) when the franchisor specifies an advertising fee \( C(A) \) for various levels of \( \gamma \). The value of \( C(A^*) \) is the solution to Program P1.

Figure 2 shows that the ratio \( \frac{C(A^*)}{C(A^0)} \) depends on \( \gamma \). As the probability of occurrence of the high demand state \( \gamma \) increases, the franchisor’s optimal advertising fee \( C(A^*) \) increases. In contrast, in Program P2, the optimal advertising royalty \( a^* \) is constant and independent of \( \gamma \). Hence, the ratio \( \frac{a^*}{a^0} \) is independent of \( \gamma \). This further illustrates that the advertising royalty is a more flexible instrument than the advertising fee.

To get a deeper understanding, we will examine the advertising expenditures for the two specifications.

We denote as \( E \), the expected advertising expenditure when the advertising royalty is specified in the contract. The expectation is
taken over the demand states such that \( E = \sum_{i=1}^{2} p_i D_i \). \( K \) denotes the advertising expenditure when an advertising fee is specified in the contract. One question is: How does \( E \) compare to \( K \)? We evaluate \( \frac{E}{K} \) as a percentage, which is displayed in Figure 3. It is interesting that the expected advertising expenditure when an advertising royalty is specified in the contract is lower than the advertising expenditure when a fee is specified in the contract. This is true for all values of \( \rho \). Next, denote \( A_1 \) and \( A_2 \) to be advertising expenditures corresponding to the high and low demand states when an advertising royalty is specified in the contract. A comparison of this with advertising expenditures when a fee is specified is also displayed in Figure 3. The advertising fee \( K \) is higher than the advertising expenditure in the low demand state \( A_2 \) for all values of \( \rho \). However, the advertising fee \( K \) is lower than the advertising expenditure in the high demand state, \( A_1 \), for \( 0 < \rho < 1 \). When \( \rho \) takes on values in the range \( 0 < \rho < 1 \), the advertising fee \( K \) is higher than the advertising expenditure in the high demand.
state \( A_1 \). In this way, we see how the advertising royalty specification enables the franchisor to adjust the level of advertising based on new information in the second stage.

3. ANALYTICAL RESULTS

We next establish the analytical results corresponding to those in the numerical example. We first solve for the second stage choices of the franchisor and franchisee and then for the first-stage choices for Programs P1, P2 and P3. To obtain the franchisee’s optimal price and service level, we differentiate the franchisee’s profit given by equations 6, 13 and 20 with respect to \( s_i \) and \( p_i \). To obtain the optimal quality level, we differentiate the franchisor’s profit given by equations 5, 12 and 19 with respect to \( q_i \).

The choice of optimal advertising under the three programs is determined as follows. For Program P1, since an advertising fee \( C(A) \) is specified, the level of advertising \( A \) in the second stage is uniquely determined. For Program P2 where an advertising royalty is specified, the actual advertising expenditures depend on the advertising royalty
payments. The relationship between $a$ and the actual advertising expenditure is somewhat complicated and is derived below. For Program P3, we differentiate the franchisor’s profit with respect to $A_i$ to obtain the optimal advertising level.

Having obtained the second stage choices of the franchisor and franchisee, we then solve for the first stage equilibrium. We will offer two propositions. The first demonstrates that it is never optimal to leave advertising unspecified. The second identifies conditions under which advertising royalty specification is superior to advertising fee specification.

### 3.1. Second Stage Nash Equilibrium

Following the approach described above, we obtain the second stage Nash Equilibrium choices under Programs P1, P2 and P3. In the interests of space, we present only the main results.

#### 3.1.1. Program P1

Let $i(q_i, p_i; r, A)$ denote the optimal service level and $p_i(q_i, s_i; r, A)$ denote the optimal price level obtained as described above.

\[
e_i(q_i, p_i; r, A) = \frac{1}{\beta} - c_i p_i \tag{26}
\]

\[
\alpha_i(q_i, s_i; r, A; \dot{\phi}_i^* + a_A + s_i - q_i) \tag{27}
\]

The franchisor’s best choice of quality denoted by $i(p_i, s_i; r, A)$ is

\[
\alpha_i(p_i, s_i; r, A) = \nu \phi_i \tag{28}
\]

We can use equations 26, 27 and 28 to solve for $p_i$ and $s_i$ in closed form for our specification. We get

\[
p_i(r, A; \nu) = \left(\frac{\phi_i^* - \nu A}{\dot{\phi}_i^*}\right) \tag{29}
\]
3.1.2. Program P2

The optimal service level $q_i(p_i; r, a)$ is

$$q_i(p_i; r, a) = \left( \frac{\sigma_i - \bar{r}_i}{2\sigma - 1} \right)$$

(30)

The optimal price level $p_i(q_i; s_i; r, a)$ is

$$p_i(q_i; s_i; r, a) = \left( 1 + \sigma_i + r_i \right)$$

(31)

The franchisor's best choice of $q_i$, denoted by $i(p_i; s_i; r, a)$ is

$$i(p_i; s_i; r, a) = \left( \frac{\sigma_i - r_i}{2\sigma} \right)$$

(32)

The optimal levels of price, service and quality depend on each other as well as on the advertising level. This interdependence is captured in our model by invoking the rational expectations equilibrium. Mathematically, this implies that the following holds as an identity in equilibrium.

$$\frac{\partial P_i}{\partial \sigma} = 0$$

(33)

The above identity states that under equilibrium, the money spent on advertising is equal to the advertising payments received by the franchisor. We can use the above identity to obtain

$$\sigma_i(p_i; s_i; r, a) = \frac{\partial P_i}{\partial \sigma}$$

(34)

The optimal levels of price, service and quality depend on each other as well as on the advertising level. This interdependence is captured in our model by invoking the rational expectations equilibrium. Mathematically, this implies that the following holds as an identity in equilibrium.

$$\frac{\partial \sigma_i}{\partial \sigma} = \frac{\partial P_i}{\partial \sigma}$$

(35)

The above identity states that under equilibrium, the money spent on advertising is equal to the advertising payments received by the franchisor. We can use the above identity to obtain

$$\frac{\partial \sigma_i}{\partial \sigma} = \frac{\partial P_i}{\partial \sigma} = \frac{\sigma_i(p_i; s_i; r, a)}{\partial \sigma}$$

(36)

$$\frac{\partial \sigma_i}{\partial \sigma} = \frac{\sigma_i(p_i; s_i; r, a)}{\partial \sigma} = \frac{\partial P_i}{\partial \sigma}$$

(37)

$$\frac{\partial \sigma_i}{\partial \sigma} = \frac{\sigma_i(p_i; s_i; r, a)}{\partial \sigma} = \frac{\partial P_i}{\partial \sigma}$$

(38)

$$\frac{\partial \sigma_i}{\partial \sigma} = \frac{\sigma_i(p_i; s_i; r, a)}{\partial \sigma} = \frac{\partial P_i}{\partial \sigma}$$

(39)
Substituting the above in equations 32 to 34, we obtain

\begin{align}
\sigma_i & = \frac{iv_i - \alpha_i \omega_i}{(2 \theta - \phi \sqrt{2 \theta \alpha_i} - \phi \sqrt{2 \theta})} \\
\epsilon_i & = \frac{\phi_i - \phi_i \sqrt{2 \theta \alpha_i}}{(2 \theta - \phi \sqrt{2 \theta \alpha_i} - \phi \sqrt{2 \theta})} \\
\eta_i(\gamma, \alpha) & = \frac{\gamma \sqrt{2 \theta \alpha_i}}{(2 \theta - \phi \sqrt{2 \theta \alpha_i} - \phi \sqrt{2 \theta})} 
\end{align}

Equations 36-38 provide closed-form solutions for the second stage choices of the franchisor and franchisee.

### 3.1.3. Program P3

The optimal service \( q_i \) is

\[ q_i = (1 - \tau) \mu \]  

And the optimal price \( \rho_i \) is

\[ \rho_i = \frac{\phi_i \alpha_i \mu_i - \phi \sqrt{2 \theta \alpha_i}}{2 \theta} \]  

The franchisor’s best choice of \( q_i \), denoted by \( q_i^* \), is

\[ q_i^* = \rho_i  \]  

Similarly, the franchisor’s best choice of advertising \( A_i \), denoted by \( A_i^* \), is

\[ A_i^* = \tau \rho_i \]  

We can use equations 39, 40, 41 and 42 to solve for \( \rho_i \), \( q_i \), \( A_i \) and \( \xi_i \) in closed form for our specification. Doing this, we obtain

\begin{align}
\sigma_i & = \frac{\gamma}{2 \theta - \phi \sqrt{2 \theta \alpha_i} - \phi \sqrt{2 \theta}} \\
\eta_i & = \frac{\rho_i \gamma}{2 \theta - \phi \sqrt{2 \theta \alpha_i} - \phi \sqrt{2 \theta}} \\
\alpha_i & = \frac{(1 - \gamma \phi_i)}{(2 \theta - \phi \sqrt{2 \theta \alpha_i} - \phi \sqrt{2 \theta})} \\
\xi_i & = \tau \rho_i \\
\lambda_i & = \frac{\gamma}{2 \theta} - \frac{\rho_i \gamma}{\alpha_i} - 1 
\end{align}
3.1.4. First Stage Decisions of Franchisor

The first stage decisions of franchisor under Programs P1, P2 and P3 can now be obtained by solving Programs P1, P2 and P3 with the second stage choices as constraints. Appendices A, B and C provide the second-stage and first-stage solutions to Programs P1, P2 and P3.

There are constraints on the parameters of the demand function that are required for the demand function to be positive under each of the Programs P1, P2 and P3. We discuss them in turn. From Appendix A, the constraint on the parameters required for the demand function under Program P1 to be positive is 
\[ 2 - \frac{\alpha}{2} > 2 \left( 1 + \frac{\alpha}{2} \right) \left( \frac{\alpha}{2} .25 \right) \] and \[ 2 > 1. \]
We refer to these constraints as Constraint 1 and 2 respectively. From Appendix B, the constraint on the parameters required for the demand function under Program P2 to be positive is \[ 2 > 2 \]. We refer to this constraint as Constraint 3. From Appendix C, the constraint on the parameters required for the demand function under Program P3 to be positive is \[ 2 > 2 + 1. \] We refer to this constraint as Constraint Next, we present the central results of the paper.

3.2. Central Results

First, we compare the channel profits from programs P1, P2 and P3.

PROPOSITION 1: A contract with a specification of advertising payment is superior to one that leaves advertising unspecified.

Proof: See Appendix D.

Discussion: We find that either an advertising royalty payment or an advertising fee payment is more profitable than using no advertising payment in the franchise contract because it allows the franchisor to commit to a specific advertising level. Funding advertising from revenues received from fixed fees, sales royalties or the mark-up from the transfer of goods is sub-optimal since advertising is hidden-action with respect to the franchisee. Therefore, a specification of advertising in the form of a fixed fee or royalty is superior. This is consistent with Mathewson and Winter (1985) and Desai (1997).

Corollary 1: If \[ \lambda = 0 \] or \[ \lambda = 1 \], a contract with advertising specified in the form of a fee is superior to a contract specified in the form of an advertising royalty.
Discussion: The advertising fee specification implies that the franchisor would have to commit to a level of advertising that is the same under both high and low demand states. Since $\gamma = 0$ or $\gamma = 1$, the franchisor does not find the need to adjust advertising based on the second stage information. Moreover, the adverse consequences of hidden actions of the franchisor are absent when the contract specifies an advertising fee. Hence, an advertising fee is preferable to an advertising royalty.

Note that the uncertainty in demand is at a maximum when $\phi = 0.5$. Corresponding to this, let

$$\Delta \Pi = \Pi_F - \Pi_R$$

where $\Pi_F$ denotes the channel profits when an advertising fee is specified, and $\Pi_R$ denotes channel profits when an advertising royalty is specified.

Appendix D provides explicit expressions for $X$, $Y$ and $Z$. Of course, if $\gamma = 1$, $\phi > 0$, as we already know from Corollary 1. The question is what happens as $\gamma - 1$ increases. This will depend on the sign of $X$, which only depends on the demand sensitivity parameters. The following proposition identifies conditions under which the advertising royalty is superior to the advertising fee.

Proposition 2: If $X > 0$, the franchisor should specify an advertising fee. If $X < 0$, there exists a $\gamma^*$ such that for $\gamma > \gamma^*$, the franchisor should specify an advertising royalty. Moreover, for every $\gamma^* > \gamma^*$, there exists a pair $(l, h)$, $0 < l < h < 1$ such that for $\gamma \in (l, h)$, a contract with advertising specified in the form of an advertising royalty is superior to a contract with advertising specified as a fee.

Proof: See Appendix D.

Discussion: This is a new result, and it provides an explanation for observed royalty provisions in franchise contracts. The franchisor can commit to advertising either in the form of an advertising fee or an advertising royalty. As discussed in Section 2.5 under the numerical
example, the intuition here is that the advertising royalty specification is a more flexible instrument than a single, fixed advertising fee since it takes into account the information in the second stage on the demand state. Of course, the advertising royalty specification has the adverse consequences of hidden action. Thus, if all contingencies affecting demand cannot be predicted at the time the parties enter into the contract, the advertising royalty specification is a more flexible instrument than an advertising fee.

3.3. Summary of Results

We have established that a contract with advertising specifications is always superior to a contract with no advertising specifications due to the commitment value of specifying advertising. Further, we have shown that the advertising fee is better than the advertising royalty in the absence of demand uncertainty because the former does not adversely affect the franchisee’s choice of price and service while the latter does. However, the advertising royalty is a flexible instrument since it allows the franchisor to take into account information available in the second stage on the state of demand.

Hence, from an empirical perspective, since demand uncertainty is usually present, we would expect advertising royalties to be specified more often. Consistent with this, we find that an advertising royalty is specified by roughly 76% of the franchises considered in Table 1a and 66% of the 54 franchises in Table 1b. Further, we expect that if the demand is highly uncertain, the advertising royalty is the preferred specification based on the results of our model. This is an important empirical issue and future research on this topic would benefit from an empirical model at the individual franchise level to test the propositions we present in this paper.

Finally, some franchisors do not use any advertising fee/royalty. While this practice does not appear to be justifiable within the results of our model, it is likely that the administrative and monitoring costs are an important factor in not specifying advertising payments. Another factor may be that certain business-format franchisors may require more local advertising rather than national advertising. This may imply that the cost of administering the advertising fund may exceed the benefits, leading to no specification of advertising (Desai, 1997).
4. CONCLUSIONS

We investigate the role of advertising payments in franchise contracts. Our analysis explains the role of advertising payments in improving channel coordination.

Our model incorporates the idea that the franchisor and franchisee are in an ongoing relationship where there is demand uncertainty. We show that specifying an advertising payment in the form of a fee or a royalty is better than no specification since it commits the franchisor to invest the payments in advertising. Further, a specification of advertising royalty is a more flexible instrument than the advertising fee since it permits advertising expenditures to be adjusted based on information not available at the time the contract is written. To do so with the fee option, a menu of fees would be needed, which is likely to be impracticable. Some franchise contracts do not specify advertising payments. This is probably due to administrative and monitoring costs involved in the use of advertising payments.

A useful direction for future research is to develop an empirical model at the individual franchise level to test the propositions we propose in this paper. Some business-formal franchises may require more local advertising while others may require more national advertising. Another avenue for future research is the issue of what is the optimal allocation between national and local advertising. Finally, future research could explain why a large number of franchisors do not use any type of advertising fee.

NOTES

1. Our model assumes that both the franchisor and franchisee are risk neutral. If, however, the franchisee is assumed to be risk averse and the franchisor risk neutral, it is not optimal to have the franchisee bear all the risk. Share contracts would then emerge as a compromise between the need to provide the franchisee with insurance and the need to motivate the franchisee.

2. In some franchise contracts, the wholesale price may also be specified by the percentage of gross margin or the net margin. In this paper, we do not consider choice of wholesale price explicitly to keep the model and analysis simple.

3. The franchisor could charge both an advertising fee and an advertising royalty, but we do not explicitly consider this for two reasons. First, a majority of the franchisors uses one but not both. Second, we want our model to reflect the idea that the number of demand states is large relative to the number of advertising levels specified in the contract. Since we have only two demand states, we restrict the franchisor to use only an advertising fee or advertising royalty.
4. In the case of advertising, an alternative specification would be to make demand a concave function of advertising, for example, GRP's, and the cost of advertising to be linear in GRP's. However, our formulation is equivalent, the results do not change and our model thus has the advantage of easier exposition.

5. We thank an anonymous referee for bringing this to our attention.

6. In fact, it can be shown that for certain ranges corresponding to high uncertainty, the specification of advertising in the form of an advertising fee is inferior to no specification. However, one form of commitment, either as an advertising royalty or an advertising fee, is always superior to no specification.

7. Readers not interested in the mathematical details may proceed directly to the propositions in Section 3.2.

8. The detailed derivations are available from the authors upon request.

REFERENCES


APPENDIX A: Program P1

The Lagrangian $L$ is given as

$$L = \sum_{\mu} \mu(\nu, \lambda) - C(\nu) - C(\lambda) - \lambda \cdot f \cdot \lambda$$

(A.1)

This problem has to be maximized with respect to $(r, A, \lambda)$. It can be solved by invoking the Kuhn-Tucker conditions for the Lagrangian $L$, given by

$$\frac{\partial L}{\partial r} = 0$$

(A.2)

$$\frac{\partial L}{\partial A} \geq 0$$

(A.3)

$$\frac{\partial L}{\partial \lambda} \leq 0$$

(A.4)

$$\frac{\partial L}{\partial \lambda} \geq 0$$

(A.5)

$$\frac{\partial L}{\partial \lambda} \leq 0$$

(A.6)

$$\frac{\partial L}{\partial \lambda} \geq 0$$

(A.7)

$$\frac{\partial L}{\partial \lambda} > 0$$

(A.8)

$$\lambda, \sum_{\mu} \mu(\nu, \lambda) \cdot f - C(\nu) - C(\lambda) = 0$$

(A.9)

$$\lambda \cdot (1 - r) = 0$$

(A.10)

$$\lambda \geq 0$$

(A.11)

$$\lambda \geq 0$$

(A.12)

In light of equation A.2, $1 = 1$ and the Lagrangian becomes

$$L = \sum_{\mu} \mu(\nu, \lambda) - C(\nu) - C(\lambda) - \lambda \cdot f \cdot \lambda$$

From the equation A.5,

$$\frac{\partial L}{\partial \lambda} = 0$$

(A.13)
Note that $r$ cannot be equal to 1 because A.9 implies
\[-\sum_{j} z_{j} C'(x_{j}) / f\]
This is not true since $f > 0$. Therefore, as long as there is a finite fixed fee in this model, $r < 1$ and $0 < r < 1$.

Multipling equation A.13 by $(1 - r)$,
\[rQ - rQ(r1 - r) \frac{\delta L}{\delta r} = r(1 - r) \sum_{j} z_{j} \left[ \frac{\partial C(x_{j})}{\partial r} - C'(x_{j}) \frac{\partial C'(x_{j})}{\partial r} \right] - \lambda_{3} (1 - r) \quad (A.14)\]

But, from equation A.10,
\[\lambda_{3} (1 - r) = 0\]

Hence, equation A.14 is now
\[rQ - rQ \frac{\delta L}{\delta r} - r(1 - r) \sum_{j} z_{j} \left[ \frac{\partial C(x_{j})}{\partial r} - C'(x_{j}) \frac{\partial C'(x_{j})}{\partial r} \right] = 0\]

This implies
\[r - 0 = 0 \quad (A.15)\]
\[r = 1 \quad (A.16)\]
\[\sum_{j} z_{j} C'(x_{j}) / \partial r = C'(x_{j}) / \partial r = 0 \quad (A.17)\]

From equation A.6,
\[d \sum_{j} z_{j} \left[ p_{j} T_{j} - C(A_{j}) - C(x_{j}) \right] = 0 \quad (A.18)\]

Now, the second stage choices of the franchisee are obtained by differentiating the franchisee's profit with respect to $p_{j}$ and $x_{j}$. The second stage choice of the franchisor is obtained by differentiating the franchisor's profit with respect to $q_{f}$.

\[
\epsilon_{j}(q_{f}, x) = \frac{[q_{j} + \alpha_{j}]}{2q_{f} - 1} \\
\gamma_{j}(q_{f}, x) = \frac{[q_{j} + \alpha_{j}]}{2q_{f} - 1} \\
\sigma_{j}(q_{f}, x) = \frac{(1 - r)[q_{j} + \alpha_{j}]}{2q_{f} - 1} 
\]
Recall that we set \( \gamma = 1 \), and hence, the results are in terms of the remaining parameters of the demand function. Further, \( \gamma \) does not appear in the demand function since \( \gamma = (1 - \theta) \). We substitute the second-stage choices in equation A.17 and A.18 and solve using the Solve equations command in Mathematica (Wolfram, 1991). The optimal first stagesolutions satify the following necessary and sufficient conditions for a maximum:

\[
\begin{align*}
\frac{\partial L}{\partial \gamma}, A' &= 0 \\
\frac{\partial L}{\partial \omega}, A' &= 0 \\
\frac{\partial^2 L}{\partial \gamma^2}, A' &\leq 0 \\
\frac{\partial^2 L}{\partial \omega^2}, A' &\leq 0
\end{align*}
\]

and the Hessian matrix given by

\[
\begin{bmatrix}
\frac{\partial^2 L}{\partial \gamma^2} & \frac{\partial^2 L}{\partial \gamma \partial \omega} \\
\frac{\partial^2 L}{\partial \omega \partial \gamma} & \frac{\partial^2 L}{\partial \omega^2}
\end{bmatrix}
\]

The Hessian matrix is negative semi-definite. Therefore, the pair \( \{r^*, A^*\} \) is the unique optimal solution.

We now state the first-stage equilibrium solutions. The optimal royalty is:

\[
r^* = \gamma
\]

and

\[
A^* = \frac{\sigma(\theta^2 - 25\mathcal{E} + \gamma(\theta - 1))}{(2\theta - 1 - \sigma^2)(\theta - 25)}
\]  
(A.19)

For the demand function to be positive, we require that \( \gamma > \frac{1}{2} \) and \( \gamma > 1 \). These are constraints 1 and 2

\[
\mathcal{E} = \frac{\gamma(\theta - 25)}{(2\theta - 1 - \sigma^2)(\theta - 25)}
\]

The franchisor’s profit in this case is

\[
\Pi^*_f = \frac{\gamma(\theta - 25)}{(2\theta - 1 - \sigma^2)(\theta - 25)}
\]

Note that \( r^* = \frac{1}{2} \) since the demand function is equally sensitive to service and quality. In general, \( r^* \) will depend on \( \theta \) and \( \sigma \), the service and quality sensitivity parameters. This is not reported here in the interests of parsimony, but the proof is available from the authors.
APPENDIX B: Program P2

The Lagrangian \( L \) is:

\[ L = \sum \left[ \gamma_i + \alpha_i \left( \beta_i - \gamma_i \right) \right] \quad \gamma_i \rightarrow \alpha_i \]

This problem has to be maximized with respect to \((r, a, f)\). It can be solved by invoking the Kuhn-Tucker conditions for the Lagrangian \( L \), given by:

\[
\begin{align*}
\frac{dL}{dr} &= 0 \\
\frac{dL}{da} &= 0 \\
\frac{dL}{df} &= 0 \\
\frac{\partial L}{\partial \lambda} &= 0 \\
\frac{\partial L}{\partial \beta_i} &= 0 \\
\frac{\partial L}{\partial \alpha_i} &= 0
\end{align*}
\]

Since the solution of the above Lagrangian is similar to that in Appendix A, we omit it here in the interests of space.\(^{10}\)

The closed-form specifications for the second stage choices of price, advertising, services and quality are:

\[ \rho(r, a) = \left( \gamma_i - \alpha_i \right) \frac{1}{\sqrt{2\pi \sigma^2}} \exp\left( -\frac{(x-a)^2}{2\sigma^2} \right) \]

\[ \beta_i = \frac{\gamma_i - \alpha_i}{\sqrt{2\pi \sigma^2}} \exp\left( -\frac{(x-a)^2}{2\sigma^2} \right) \]

\[ \alpha_i = \frac{(1-r)\gamma_i \sqrt{2\pi \sigma^2}}{\sqrt{2\pi \sigma^2} - \alpha_i} \]

\[ \tau_i = \frac{r\gamma_i \sqrt{2\pi \sigma^2}}{\sqrt{2\pi \sigma^2} - \alpha_i} \]

\(^{10}\)This is available upon request from the authors.
From the equations B.5 and B.6, we have

\[ \sum_{y_{i}} \frac{\mu \pi_{i}}{\beta} (\beta_{i} - \gamma_{i}) \leq 0 \]  

and

\[ \sum_{y_{i}} \frac{\mu \pi_{i} \beta_{i}}{\beta} (\beta_{i} - \gamma_{i}) \leq 0 \]  

As in Program P1, we substitute the second stage-choices in equations B.13 and B.14 and solve using the Solve equations command in Mathematica. The optimal first stage solutions satisfy the necessary and sufficient conditions for a maximum. These conditions are similar to those derived in Appendix A and are omitted here in the interest of space.

We now state the first-stage equilibrium solutions. The optimal sales royalty and advertising royalty are:

\[ r^{*} = \frac{1}{\alpha} \]  

and

\[ \sigma^{*} = \frac{\alpha \pi}{2g} \]  

The franchisor’s profit in this case is

\[ \pi^{*} = \frac{4(1 - g^{2})g^{2} \gamma + 4g^{2} \gamma}{4g^{2} - 1 - g^{2}} \]  

The constraint \( 2 > \beta \), referred to as Constraint 3, should be satisfied if the demand function under Program P2 is to be positive.

APPENDIX C: Program P3

In the case with advertising unspecified, the Lagrangian \( L \) is given as

\[ \tilde{L} = \sum_{y_{i}} \left( p_{i} \pi_{i} - \gamma_{i} \right) - \lambda_{i} \left( r_{i} - \beta_{i} \right) - r_{i} \left( \gamma_{i} - \beta_{i} \right) \]  

This problem, which is to be maximized with respect to \( (r, f) \), can be invoking the Kuhn-Tucker conditions for the Lagrangian \( \tilde{L} \), given by:

\[ \frac{\partial \tilde{L}}{\partial f} = 0 \]
Since the solution of the above Lagrangian is similar to that in Appendix A, we omit it here in the interest of space. 11

The closed-form specifications for the price, quality, service, and advertising are as given:

\[ p_i^* = \frac{q_i}{2b - \alpha_i r_i - 1} \]  
\[ q_i^* = \frac{r_i \alpha_i}{2a} \]  
\[ r_i^* = \left[ \frac{(1 - \alpha_i) p_i}{2b - \alpha_i r_i - 1} \right] \]  
\[ o_i^* = \left[ \frac{r_i \alpha_i}{2b - \alpha_i r_i - 1} \right] \]

As in Program P1 and P2, we substitute the second-stage choices in equation A.1 and solve using the Solve equations command in Mathematica. The optimal first-stage solutions satisfy the necessary and sufficient conditions for a maximum. We now state the first-stage equilibrium solutions. The optimal sales royalty is:

\[ R = \frac{2 \bar{q} + 2a \bar{q}^2 \bar{a} - \bar{a}^2 - 1}{2(2\bar{q} + 1) \bar{a}^2 - \bar{a}^2 - 1} \]  
\[ T = \frac{[6 - 2\bar{q}^2 \bar{a} - 1 - \bar{a}^2 + 2(2\bar{q} + 1)] \bar{a} \bar{q} - \bar{q}^2}{2(2\bar{q} + 1) \bar{a}^2 \bar{q} - \bar{q}^2 \bar{a}} \]

The constraint \( 2 > \bar{a} + 1 \), referred to as Constraint 4, should be satisfied if the demand function under Program P3 is to be positive.

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11 This is available upon request from the authors.
APPENDIX D: Proof of Proposition 1

To prove Proposition 1, it is sufficient to show that the channel profit under Program P3 is less than the maximum from either Program P2 or Program 1. So, we show that channel profit under Program 3 is always less than that under Program P2.

\[ \Pi_{P3} - \Pi_{P2} = \frac{\alpha^2 (\beta^2 - 1)(1 - 2\rho'(\theta') (1 + \mu(\theta') - 1)^2)}{\alpha^2 \beta^2 (1 + 2\rho'(\theta') (1 + \mu(\theta') - 1)^2)} > 0 \]  \hspace{1cm} (D.1)

The above is true if \( \theta' > 1 \) or if Constraint 2 is satisfied. Hence, Proposition 1 holds the demand function is positive. Q.E.D.

5.4.1 Proof of Corollary

We compare Program P1 (or, where advertising is specified in the form of an advertising fee) with the Program P2 (or, where advertising is specified in the form of an advertising royalty). Consider the case where \( \theta = 0 \).

\[ \Pi_{P1} - \Pi_{P2} = \frac{\alpha^2 (\beta^2 - 1)(1 - 2\rho'(\theta') (1 + \mu(\theta') - 1)^2)}{\alpha^2 \beta^2 (1 + 2\rho'(\theta') (1 + \mu(\theta') - 1)^2)} > 0 \]  \hspace{1cm} (D.2)

The above is true if \( \theta' > (2 + \theta')(0.25) \) or if the Constraint 1 holds.

\[ \Pi_{P1} - \Pi_{P2} = \frac{\alpha^2 (\beta^2 - 1)(1 - 2\rho'(\theta') (1 + \mu(\theta') - 1)^2)}{\alpha^2 \beta^2 (1 + 2\rho'(\theta') (1 + \mu(\theta') - 1)^2)} > 0 \]  \hspace{1cm} (D.3)

Consider the case where \( \theta = 1 \).

Again, the above is true if \( \theta' > (2 + \theta')(0.25) \) or if Constraint 1 holds. Q.E.D.

5.4.2 Proof of Proposition 2

We now compare Program P1, where advertising is specified in the form of a fee, with Program P2, where advertising is specified in the form of an advertising royalty.

\[ \Delta \Pi = \Pi_{P1} - \Pi_{P2} = X\omega^2 + Y\omega + Z \]  \hspace{1cm} (D.4)

Where \( \theta = 1 \) and \( X, Y, \) and \( Z \) are:

\[ X = \frac{\alpha^2 (\beta^2 - 1)(1 - 2\rho'(\theta') (1 + \mu(\theta') - 1)^2)}{\alpha^2 \beta^2 (1 + 2\rho'(\theta') (1 + \mu(\theta') - 1)^2)} \]  \hspace{1cm} (D.5)
Note that the uncertainty is at a maximum when $1 = 1/2$. Setting $1 = 1/2$, $Y$ and $Z$ are:

$$Y = \frac{\mu (\theta - \bar{\theta}) (\theta - \bar{\theta})}{(3\theta - 1)^2 \beta^2 \left[ 2 + \alpha^2 \right]}$$

and

$$Z = \frac{2(\theta - \bar{\theta}) (\theta - \bar{\theta})}{(3\theta - 1)^2 \beta^2 \left[ 2 + \alpha^2 \right]}$$

We can see that $Y = Z > 0$ provided $2 > \left[ (2 + \frac{\alpha^2}{2}) \left( \frac{1}{2} - 0.25 \right) \right]$ and $2 > 1 + \frac{\alpha^2}{2}$ or if Constraints 1 and 4 hold.

1. If $X > 0$, then for all positive, $> 0$.

2. If $X < 0$, equation D.4 has one positive root and one negative root. In other words, there exists a $*$ such that for $> *$, the franchisor should specify royalty.

Since $> 0$ at $1 = 0$ and $> 0$ at $1 = 1$, by continuity arguments, for every $< *$, there exists a pair $(l, h)$, $0 < l < h < 1$ such that for $1 \epsilon (l, h) < 0$. Q.E.D.